www.ijiarec.com

ISSN:2348-2079

Volume-5 Issue-2

International Journal of Intellectual Advancements and Research in Engineering Computations

Weakly Generalized Fuzzy Closed sets and Maps in Ŝostak's Fuzzy Topoloical Spaces

R. V. M. Rangarajan *, M. Chandrasekharan † and A. Vadivel ‡

Abstract

In this paper, we introduce and study the concept of r -weakly generalized fuzzy closed sets in ŝ ostak's fuzzy topological spaces. Fuzzy WG-connectedness is introduced and studied with help of r -weakly generalized fuzzy closed sets, Fuzzy weakly generalized continuity and Fuzzy weakly generalized irresolute mappings are introduced and the relationship between these mappings and other mappings introduced previously are investigated. Also, some separation axioms of r -weakly generalized fuzzy closed sets are introduced and studied.

Keywords and phrases: r-weakly generalized fuzzy closed sets, r-generalized fuzzy weakly closed sets, Fuzzy WG-connected (Fuzzy GW-connected) sets, Fuzzy weakly generalized continuity, generalized fuzzy weakly continuity, FWG-irresolute, GFW-irresolute, and r-FWG-regular (r-FWG-normal) spaces.

AMS (2000) subject classification: 54A40.

1 Introduction and preliminaries

Kubiak [18] and ŝostak [24] introduced the fundamental concept of a fuzzy topological structure, as an extension of both crisp topology and fuzzy topology [3], in the sense that not only the objects are fuzzified, but also the axiomatics. In [25, 26], ŝostak gave some rules and showed how such an extension can be realized. Chatopadhyay et al., [4] have redefined the same concept under the name gradation of openness. A general approach to the study of topological type structures on fuzzy power sets was developed in [[10]-[12], [18, 19]].

Weakly closed sets, weakly continuous mappings were introduced and investigated by [20]. The concept of g -closed sets was also considered by Dunham [7] in 1982 and by Dunham and Levine [6] in 1980. J. Mahanta and P. K. Das used w-closed sets to define and investigate the notion of generalized weakly closed sets using weakly-closure operator. Balachandran et al., [2] in 1991, defined a new class of mappings called generalized continuous (g -continuous, for short) mappings which contains the class of continuous mapping.

---

*Department of Mathematics, K. S. R. College of Engineering, Tiruchengode, rangarajan.rvm@gmail.com †Dean of Science and Humanities, Nanda Engineering College, Erode-638052, mcbrinda@gmail.com ‡Department of Mathematics, Annamalai University, Annamalainagar, Tamil Nadu-608 002, avmaths@gmail.com.

Later in 2007, El. Shafei and Zakari [8] have introduced and study some generalizations of fuzzy continuous mappings.

The aim of this paper is to define the concepts of r-weakly generalized fuzzy closed sets in ŝostak fuzzy topological spaces. Fuzzy WG-connectedness is introduced and studied with help of r-weakly generalized fuzzy closed sets, fuzzy weakly generalized continuity and fuzzy weakly generalized irresolute mappings are introduced and the relationship between these mappings and other mappings introduced previously are investigated. Also, some separation axioms of r-weakly generalized fuzzy closed sets are introduced and studied.

Throughout this paper, nonempty sets will be denoted by X, Y etc., $I = [0, 1]$ and $I_0 = (0, 1]$. For $\alpha \in I$, $\bar{\alpha}(x) = \alpha$ for all $x \in X$. A fuzzy point xt for $t \in I_0$ is an element of I^X such that $(y) =\{t$ if $y = x$ 0 if $y \notin x$. The set of all fuzzy points in X is denoted by $P\mathrm{t}(X)$. A fuzzy point $x_t \in \lambda$ iff $t < (x)$. A fuzzy set λ is quasi-coincident with μ, denoted by $\lambda q\mu$, if there exists $x \in X$ such that $(x) + (x) > 1$. If λ is not quasi-coincident with μ, we denoted $\lambda\bar{q}\mu$. If $A \subset X$, we define the characteristic function χ_A on X by $\chi_A(x) =\{1$ if $x \in A$, 0 if $x \notin A$. All other notations and definitions are standard, for all in the fuzzy set theory.

Lemma 1.1. [13] Let X be a nonempty set and $\lambda, \mu \in I^X$. Then

(i) $\lambda q\mu$ iff there exists $x_t \in \lambda$ such that $x_t\, q\mu$.

(ii) $\lambda q\mu$, then $\lambda \wedge \mu \neq \underline{0}$.

(iii) $\lambda\bar{q}\mu$ iff $\lambda \leq \underline{1} - \mu$.

(iv) $\lambda \leq \mu$ iff $x_t \in \lambda$ implies $x_t \in \mu$ iff $x_t\, q\lambda$ implies $x_t\, q\mu$ implies $x_t\, \bar{q}\lambda$.

(v) $x_t\ \bar{q} \vee \mu i$ iff there exists $i_0 \in \Lambda$ such that $x_t\, \bar{q}\mu_{i0}$.

Definition 1.1. [22, 24] A function: $I^X \to I$ is called a fuzzy topology on X if it satisfies the following conditions:

(O1) $\tau(\underline{0}) = \tau(\underline{1}) = 1$,

(O2) $\tau(\vee_{i\in\Gamma}\ \mu_i) \geq \wedge_{i\in\Gamma}\ \tau(\mu_i)$, for any $\{\mu_i\}_{i\in\Gamma} \subset I^X$,

(O3) $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu 1) \wedge \tau\ (\mu 2)$, for any $\mu_1, \mu_2 \in I^X$.

The pair (X, τ) is called a fuzzy topological space (for short, fts).

Remark 1.1. [16] Let (X, τ) be a fuzzy topological space. Then, for each $r \in I_0$, $\tau_r = \{\mu \in I^X\colon (\mu) \geq r\}$ is a Change's fuzzy topology on X.

Theorem 1.1. ([5]) Let (X, τ) be a fts. Then for each $\lambda \in I^X$, $r \in I_0$ we define an operator $C_\tau\colon I^X \times I_0 \to I^X$ as follows: $(\lambda, r) = \wedge\{\mu \in I^X : \lambda \leq \mu, \tau(\underline{1} - \mu) \geq r\}$. For $\lambda, \mu \in I^X$ and $r, \in I_0$, the operator $C\tau$ satisfies the following conditions:

(1) $C_\tau(0,r) = 0$,

(2) $\lambda \leq C_\tau(\lambda, r)$,

(3) $C_\tau(\lambda, r) \vee C\tau(\mu, r) = C\tau(\lambda \vee \mu, r)$,

(4) $C_\tau(\lambda, r) \leq C\tau(\lambda, s)$ if $r \leq s$,

(5) $C_\tau(C\tau(\lambda, r), r) = C\tau(\lambda, r)$.

Theorem 1.2. [23] Let (X, τ) be a fts. Then for each $r \in I_0$, $\lambda \in I^X$ we define an operator $I\tau\colon I^X \times I_0 \to I^X$ as follows: $(\lambda, r) = \vee\ \{\mu \in IX\colon \lambda \geq \mu, \tau\ (\mu) \geq r\}$.

For $\lambda, \mu \in I^X$ and $r, \in I_0$, the operator $I\tau$ satisfies the following conditions:

(1) $I\tau\ (1,) = 1$,

(2) $\lambda \geq I\tau\ (\lambda, r)$,

(3) $I\tau\ (\lambda, r) \wedge (\mu, r) = (\lambda \wedge \mu, r)$,

(4) $I\tau\ (\lambda, r) \leq (\lambda, s)$ if $s \leq r$,

(5) $I\tau\ ((\lambda, r), r) = (\lambda, r)$,

(6) $I\tau\ (\underline{1} - \lambda,) = \underline{1} - (\lambda, r)$ and

$(\underline{1} - \lambda,) = \underline{1} - I\tau(\lambda, r)$

Definition 1.2. [17] A fuzzy subset λ of a fts (X, τ) is said to be r-generalized fuzzy closed (r-gfc, for short) set if $C(\lambda, r) \leq \mu$ whenever $\lambda \leq \mu$, $\tau(\mu) \geq r$ and $r \in I_0$. The complement of a r-gfc set is called r-generalized fuzzy open (r-gfo, for short) set.

Definition 1.3. [16, 23] Let (X, τ) be a smooth fuzzy topological space. Then for each $\lambda, \mu \in I^X$ and $r \in I_0$,

(i) λ is called r-fuzzy semi-open (r-fso, for short) if $\lambda \leq C\tau\ ((\lambda, r), r)$.

(ii) λ is called r-fuzzy semi-closed (r-fsc, for short) if $((\lambda, r), r) \leq \lambda$.

(iii) The r-fuzzy semi-interior of λ, denoted by $SI_\tau\ (\lambda, r)$ is defined by:

$(\lambda, r) = \vee\ \{\mu \in I^X: \mu \leq \lambda, \mu$ is a r-fso$\}$.

(iv) The r-fuzzy semi-closure of λ, denoted by (λ, r) is defined by:

$(\lambda, r) = \wedge\ \{\mu \in I: \mu \geq \lambda, \mu$ is a r-fsc$\}$.

Definition 1.4. [9] Let (X, τ) be a smooth fuzzy topological space. $\lambda \in I^X$ and $r \in I_0$, λ is called a r-fuzzy weakly closed (r-fwc, for short) set if $(\lambda, r) \leq \mu$ whenever $\lambda \leq \mu$ and is r-fuzzy semiopen. Then complement of a r-fwc set is said to be a r-fuzzy weakly open (r-fwo, for short) set.

Definition 1.5. [9] Let (X, τ) be a smooth fuzzy topological space. $\lambda \in I^X$ and $r \in I_0$,
(i) $(\lambda, r) = \vee\ \{\mu \in IX: \mu \leq \lambda, \mu$ is a r-fwo set$\}$ is called the r-fuzzy w-interior of λ.
(ii) $W\ C\tau\ (\lambda, r) = \wedge\ \{\mu \in IX: \mu \geq \lambda, \mu$ is a r-fwc set$\}$ is called the r-fuzzy w-closure of λ.

Definition 1.6. [9] Let (X,τ) be a smooth fuzzy topological space. For $\lambda \in I^X$ and $r \in I_0$,

(1) λ is called r-generalized fuzzy weakly closed (r-gfwc, for short) set if $WC_\tau\ (\lambda, r) \leq \mu$ whenever $\lambda \leq \mu$ and μ is r-fuzzy weakly open set.

(2) λ is called r-generalized fuzzy weakly open (r-gfwo, for short) set if $\mu \leq WC_\tau\ (\lambda, r)$ whenever $\mu \leq \lambda$ and μ is r-fuzzy weakly closed set.

(3) The r-generalized fuzzy weakly closure of λ, denoted by $GWC_\tau(\lambda, r)$ is defined by: $GWC\tau(\lambda, r) = \wedge\{\mu \in I^X : \mu \geq \lambda$, μ is r-gfwc $\}$.

(4) The r-generalized fuzzy weakly interior of λ, denoted by (λ, r) is defined by:

$GWI\tau\ (\lambda, r) = \vee\{\mu \in I^X : \mu \leq \lambda, \mu$ is r-gfwo $\}$.

Definition 1.7. [24] Let (X, τ_1) and (Y, τ_2) be a smooth fuzzy topological spaces. Let: $X \rightarrow Y$ be a mapping.

(1) F -continuous if $\tau_2\ (\mu) \leq \tau_1\ (f^{-1}\ (\mu))$ for each $\mu \in I^Y$.

(2) F -open if $\tau_1\ (\lambda) \leq \tau_2\ ((\lambda))$ for each $\lambda \in I^X$.

(3) F -closed if $\tau_1\ (1 - \lambda) \leq \tau_2\ (1 - f\ (\lambda))$ for each $\lambda \in I^X$.

Definition 1.8. [17, 21]

Let: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be a mapping and $r \in I_0$. Then f is called:

(i) GF -continuous (resp. GF -irresolute) if $f^{-1}(\mu)$ is r-gfo for each $\mu \in I^Y$, $_2\ (\mu) \geq r$ (resp. μ is r-gfo).
(ii) GF -open (resp. GF-irresolute open) if $\tau(\lambda)$ is r-gfo for each $\lambda \in I^X$, $\tau_1\ (\lambda) \geq r$ (resp. λ is r-gfo).
(iii) GF -closed (resp. GF-irresolute closed) if (λ) is r-gfc for each $\lambda \in I^X$, $\tau_1\ (1 - \lambda) \geq r$ (resp. λ is r-gfc).

Definition 1.9. [10] Let: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be a mapping and $r \in I_0$. Then f is called:

(i) FW-continuous (resp. FW -irresolute) if $f^{-1}(\mu)$ is r-fwo for each $\mu \in I^Y$, $\tau_2\ (\mu) \geq r$ (resp. μ is r-fwo).
(ii) FW -open (resp. FW -irresolute open) if (μ) is r-fwo for each $\lambda \in I^X$,

$\tau_1(\lambda) \geq r$ (resp. λ is r-fwo).

(iii) FW-closed (resp. FW -irresolute closed) if (μ) is r-fwc for each $\lambda \in I^X$,
$\tau_1(\lambda) \geq r$ (resp. λ is r-fwc).

Definition 1.10. [14] A fts (X, τ) is said to be

(i) r-FR_0 iff $x_t\overline{q}C_\tau$ (y_s, r) implies $y_s\overline{q}C\tau(x_t, r)$ for any distinct fuzzy points $x_t, y_s \in Pt(X)$.

(ii) r-FR_1 iff $x_t\overline{q}C_\tau$ (y_s, r) implies that there exist $\mu_i \in I^X$ with $(\mu_i) \geq r$ for $i \in \{1, 2\}$ such that $x_t \in \mu_1$, $y_s \in \mu_2$ and $\mu_1\ \overline{q}\mu_2$ for any distinct fuzzy points $x_t, y_s \in (X)$.

(iii) r-FR_2 iff (or r-fuzzy regular) $x_t\overline{q}\lambda$ with $(\underline{1}-\lambda) \geq r$ implies that there exist $\mu_i \in I^X$ with $(\mu_i) \geq r$ for $i \in \{1, 2\}$ such that $x_t \in \mu_1$, $\lambda \leq \mu_2$ and $\mu_1\ \overline{q}\mu_2$.

(iv) r-FR_3 iff (or r-fuzzy normal) $\lambda_1\ \overline{q}\lambda_2$ with $(\underline{1}-\lambda_i) \geq r$ for $i \in \{1, 2\}$ implies that there exist $\mu_i \in I^X$ with $(\mu_i) \geq r$ for $i \in \{1, 2\}$ such that $\lambda_i \in \mu_i$ and $\mu_1\overline{q}\mu_2$.

Definition 1.11. [14] A fts (X, τ) is said to be

(i) r-FT_1 iff $(\underline{1} - xt) \geq r$ for each $xt \in (X)$.

(ii) r-$FT2$ iff $x_t\overline{q}y_s$ implies that there exist $\mu i \in I^X$ with $(\mu_i) \geq r$ for $i \in \{1, 2\}$
such that $x_t \in \mu_1$, $y_s \leq \mu_2$ and $\mu_1\overline{q}\mu_2$.

(iii) r-$FT_2\frac{1}{2}$ iff $x_t\ \overline{q}y_s$ implies that there exist $\mu_i \in I^X$ with $(\mu_i) \geq r$ for $i \in \{1, 2\}$
such that $x_t \in \mu_1$, $y_s \in \mu_2$ and $C\tau(\mu_1, r)\ \overline{q}C\tau(\mu_2, r)$.

(iv) r-FT_4 iff it is r-FR_3 and r-FT_1.

Definition 1.12. [15] A fts (X, τ) is said to be

(i) r-$FT_{\frac{1}{2}}$ if $(\underline{1} - \lambda) \geq r$ for each r-gfc set $\lambda \in I^X$ and $r \in I_0$.

(ii) r-GFR_2 iff $x_t\ \overline{q}\lambda$ for each r-gfc $\lambda \in I^X$ implies that these exist $\mu_i \in I^X$ with $(\mu_i) \geq r$ for $i \in \{1, 2\}$ such that $x_t \in \mu_1$, $\lambda \leq \mu_2$ and $\mu_1\ \overline{q}\mu_2$.

(iii) r-GFR_3 if $\lambda_1\ \overline{q}\lambda_2$ for each r-gfc sets $\lambda_i \in I^X$ and $i \in \{1, 2\}$ implies that there exist $\mu_i \in I^X$ with $\tau(\mu_i) \geq r$ such that $\lambda_i \leq \mu_i$ and $\mu_1\ \overline{q}\mu_2$.

Definition 1.13. [1] Let (X, τ) be a fts and $\lambda, \mu \in I^X$, $r \in I_0$. Then,

(i) Two fuzzy sets λ and μ are said to be r-fuzzy separated iff $\lambda\overline{q}C_\tau$ (μ, r) and $\mu\overline{q}C\tau$ (λ, r).

(ii) A fuzzy set which cannot be expressed as the union of two r-fuzzy separated sets is said to be r-fuzzy connected set.

Theorem 1.3. [15] Let (X, τ) be fts and $r \in I_0$.

(a) The following statements are equivalent

(i) (X, τ) is r-FR_0.

(ii) If $x_t\overline{q}\lambda$ for $(\underline{1} - \lambda) \geq r$, we have $(x_t, r)\overline{q}\lambda$.

(iii) x_t is r-gfc for each $x_t \in (X)$.

(b) r-FT_4 ⇒ r-FT_3 ⇒ r- $FT_2\frac{1}{2}$ ⇒ r-FT_2 ⇒ r-FT_1.

(c) If a fts (X, τ) is r-FR_0 and X is a finite set, then every fuzzy set $\lambda \in I^X$ is r-gfc and r-gfo.

2 r-Weakly generalized fuzzy closed sets

Definition 2.1. Let (X, τ) be a smooth fuzzy topological space. For $\lambda, \mu \in I^X$ and $r \in I_0$,

(1) λ is called r-weakly generalized fuzzy closed (r-wgfc, for short) set if $(\lambda, r) \leq \mu$ whenever $\lambda \leq \mu$ and $\tau(\mu) \geq r$.

(2) λ is called r-weakly generalized fuzzy open (r-wgfo, for short) set if $\mu \leq W\ C\tau\ (\lambda, r)$ whenever $\mu \leq \lambda$ and $(1 - \mu) \geq r$.

(3) The r-weakly generalized fuzzy closure of λ, denoted by (λ, r) is defined by: $WGC\tau\ (\lambda, r) = \wedge \{\mu \in IX: \mu \geq \lambda, \mu \text{ is } r\text{-wgfc}\}$.

(4) The r-weakly generalized fuzzy interior of λ, denoted by (λ, r) is defined by: $W(\lambda, r) = \vee\{\mu \in IX : \mu \leq \lambda, \mu \text{ is } r\text{-wgfo} \}$. From the above definitions it is clear that the following implications are true:

$$\begin{tikzcd}
\textbf{r – gfc} \arrow[r, phantom, "\nleftrightarrow"] \arrow[d, shift right] & \textbf{r - fwc} \arrow[d, shift left] \\
\textbf{r – wgfc} \arrow[u, shift right, "/" marking] \arrow[r, shift right, "/" marking] & \textbf{r - gfwc} \arrow[l, shift right] \arrow[u, shift left, "/" marking]
\end{tikzcd}$$

Remark 2.1. The intersection of two r-gfwc (resp. r-wgfc) sets is not r-gfwc (resp. r-wgfc) set, in general and the union of two r-gfwo (resp. r-wgfo) sets is not r-gfwo (resp. r-wgfo) set, in general, as shown by Example 2.3(1). Also the intersection of two r-gfwo (resp. r-wgfo) sets is not r-gfwo (resp. r-wgfo) set, in general and the union of two r-gfwc (resp. r-wgfc) sets is not r-gfwc (resp. r-wgfc) set, in general, as shown by Example 2.3(3).

The following propositions are easily proved,

Proposition 2.1. Let (X, τ) be fts, $\lambda \in IX$ and $r \in I_0$. The following statements hold

(1) If λ is r-gfwc (resp. r-wgfc), then $\lambda = GWC\tau\ (\lambda, r)$ (resp. $\lambda = (\lambda, r)$).

(2) If λ is r-gfwo (resp. r-wgfo), then λ = GWIτ(λ,r) (resp. λ= WGIτ(λ,r)).

(3) $GWC\tau\ (\underline{1} - \lambda, r) = \underline{1} - (\lambda, r)$ (resp. $WG(\underline{1}-\lambda, r) = \underline{1} - WGI\tau(\lambda, r)$).

(4) $GWI\tau\ (\underline{1} - \lambda, r) = \underline{1} - (\lambda, r)$ (resp. $W(\underline{1} - \lambda, r) = \underline{1} - WGC\tau(\lambda, r)$).

(5) $WC\tau\ (\lambda, r)$ is r-gfwc (resp. r-wgfc).

(6) GWI$\tau\ (\lambda, r)$ is r-gfwo (resp. r-wgfo).

(7) $\lambda \leq WGC\tau\ (\lambda, r) \leq GWC\tau\ (\lambda, r) \leq WC\tau\ (\lambda, r) \leq C\tau\ (\lambda, r)$.

(8) $I\tau\ (\lambda, r) \leq (\lambda, r) \leq (\lambda, r) \leq WGI\tau(\lambda, r) \leq \lambda$.

Proposition 2.2. Let (X, τ) be fts, $\lambda, \in I^X$ and $r, \in I_0$. The following statements hold

(1) $GWC\tau\ (\underline{0}, r) = 0$ (resp. $(\underline{0}, r) = \underline{0}$).

(2) $GWC\tau\ (\lambda, r) \leq GWC\tau\ (\lambda, s)$ (resp.$WGC\tau\ (\lambda, r) \leq WGC\tau\ (\lambda, s)$) if $r \leq s$.

(3) $GWC\tau\ (\lambda, r) \leq G(\mu, r)$ (resp.$WGC\tau(\lambda, r) \leq WGC\tau(\mu, r)$) if $\lambda \leq \mu$.

(4) $GWC\tau\ (\lambda, r) \vee GWC\tau\ (\mu, r) \leq GWC\tau\ (\lambda \vee \mu, r)$.

(5) $WGC\tau\ (\lambda, r) \vee WGC\tau\ (\mu, r) \leq WGC\tau\ (\lambda \vee \mu, r)$.

(6) $GWC\tau\ (GWC\tau\ (\lambda, r), r) = G(\lambda, r)$.

(7) $WGC\tau\ (WGC\tau\ (\lambda, r), r) = W(\lambda, r)$.

Example 2.1 Let $X = \{a, b\}$ and define fuzzy topology: $I^X \longrightarrow I$ as follows:

$$\tau(\lambda) = \begin{cases} 0 & if\ \lambda \in \{\underline{0},\ \underline{1}\}, \\ \frac{1}{2} & if\ \lambda = \{\underline{0.1}\,, \underline{0.3}\} \\ 0 & otherwise \end{cases}$$

In (X, τ) $\underline{0.2}$ is $\frac{1}{2}$ -wgfc ($\frac{1}{2}$ -gfwc) but it is neither $\frac{1}{2}$-gfc nor $\frac{1}{2}$-fwc.

Example 2.2 Let $X = \{a, b\}$ and $\mu \in I^X$ defined as follows: $(a) = 0.5$, $(b) = 0.2$, define fuzzy topology: $I^X \longrightarrow I$ as follows:

$$\tau(\lambda) = \begin{cases} -1 & if\ \lambda \in \{\underline{0},\ \underline{1}\}, \\ \frac{1}{2} & if\ \lambda = a_{0.5} \vee b_{0.0} \\ \frac{2}{3} & if\ \ \lambda = a_{0.5} \vee b_{0.0}\,, \\ \frac{2}{3} & if\ \lambda = \{a_{0.5} \vee b_{0.4}\,, a_{0.5} \vee b_{0.6}\} \\ 0 & otherwise \end{cases}$$

1 if λ ∈ {0, 1}, 1 2 if $\lambda = a0.5 \vee b0.0$, 2 3 if $\lambda = a0.0 \vee b0.4$, 2 3 if $\lambda \in \{a0.5 \vee b0.4, a0.5 \vee b0.6\}$, 0 otherwise.

In (X, τ) μ is 1 2 -wgfc but not $\frac{1}{2}$-gfwc.

Definition 2.2. Let (X, τ) be fts, and $\lambda, \in I^X$, $r \in I_0$. Then,

(1) Two fuzzy sets λ and μ are said to be r-fuzzy GW-separated iff $\lambda\overline{q}G$WCτ(μ, r) and $\mu\overline{q}GWC\tau(\lambda, r)$.

(2) Two fuzzy sets λ and μ are said to be r-fuzzy WG-separated iff (μ, r) and $\mu q W(\lambda, r)$.

(3) A fuzzy set which cannot be expressed as the union of two r-fuzzy GW- separated sets is said to be r-fuzzy GW -connected set.

(4) A fuzzy set which cannot be expressed as the union of two r-fuzzy WG-separated sets is said to be r-fuzzy WG-connected set.

From the above definitions it is clear that the following implications are true: r-fuzzy separated ⇒ r-fuzzy GW -separated ⇒ r-fuzzy WG-separated ⇒ $r$fuzzy WG-connected ⇒ r-fuzzy GW -connected ⇒ r-fuzzy connected.

Remark 2.2. The converse of the above implications is not true in general as shown by the following example.

Example 2.4 Let $X = \{a, b, c\}$ and $\mu, \mu_1, \mu_2, \mu_3, \mu_4 \in I^X$ defined as follows: $\mu(a) = 0.6$, $\mu(b) = 0.2$, $\mu(c) = 0.4$; $\mu_1(a) = 0.0$, $\mu_1(b) = 0.6$, $\mu_1(c) = 0.0$; $\mu_2(a) = 0.0$, $\mu_2(b) = 0.0$, $\mu_2(c) = 0.3$; $\mu_3(a) = 0.5$, $\mu_3(b) = 0.6$, $\mu_3(c) = 0.0$; $\mu_4(a) = 0.5$, $\mu_4(b) = 0.0$, $\mu_4(c) = 0.5$ Define fuzzy topology $\tau : I^X \rightarrow I$ as follows:

$$\tau(\lambda) = \begin{cases} 1 & if\ \lambda \in \{\underline{0},\ \underline{1}\}, \\ \frac{1}{2} & if\ \lambda = \mu, \\ 0 & otherwise \end{cases}$$

(1) Since GWCτ $(\mu_1, \frac{1}{2}) = \mu_1$ and $G(\mu_2, \frac{1}{2}) = \mu_2$. Then $\mu_1\overline{q}G$W$C\tau$ $(\mu_2, \frac{1}{2})$ and $\mu_2\overline{q}GWC\tau$ $(\mu_1, \frac{1}{2})$. Hence μ_1 and μ_2 are $\frac{1}{2}$ -fuzzy GW -separated. But $(\mu_1, \frac{1}{2}) = (\mu_2, \frac{1}{2}) = 1 - \mu$. So $\mu_1(\mu_2, \frac{1}{2})$ and $\mu 2qC\tau(\mu 1, \frac{1}{2})$. Thus μ_1 and μ_2 are not $\frac{1}{2}$ -fuzzy separated.

(2) Similarly, μ_3 and μ_4 are $\frac{1}{2}$ -fuzzy WG-separated. But μ_3 and μ_4 are not $\frac{1}{2}$ fuzzy GW -separated.

(3) Since $\rho 1 = \mu 1 \vee \mu 2$ where $\mu 1$ and $\mu 2$ are $\frac{1}{2}$-fuzzy GW -separated, then ρ_1 is not $\frac{1}{2}$ -fuzzy GW -connected. We show that ρ_1 is $\frac{1}{2}$ -fuzzy connected. In fact, let $\rho_1 = \eta \vee \omega$, where $\eta, \omega \in I^X - \{\underline{0}\}$. Then either $\eta\ (b) = 0.6$ or $\omega\ (b) = 0.6$. If $\eta\ (b) = 0.6$ then Cτ$(\omega, \frac{1}{2}) = \underline{1} - \mu$. So, ηqCτ$(\omega, \frac{1}{2})$. If $\omega\ (b) = 0.6$, similarly wqCτ $(\eta, \frac{1}{2})$. Thus η and ω cannot be $\frac{1}{2}$ -fuzzy connected. Hence ρ_1 is $\frac{1}{2}$ -fuzzy connected.

(4) Similarly, $\rho_2 = \mu_3 \vee \mu_4$ is $\frac{1}{2}$ -fuzzy GW -connected. But ρ_2 is not $\frac{1}{2}$-fuzzy WG-connected.

Theorem 2.1. Let (X, τ) be a fts, and $\lambda, \in I^X$, $r \in I_0$.

(1) If λ, μ are r-fuzzy GW -separated and $\gamma, \eta \in I^X - \{\underline{0}\}$ such that $\gamma \leq \lambda$ and $\eta \leq \mu$, then γ, η are also r-fuzzy GW-separated.

(2) If $\lambda\overline{q}\mu$ and either both are r-gfwo or both r-gfwc, then λ and μ are r-fuzzy GW -separated.

(3) If λ, $\mu \in I^X - \{\underline{0}\}$ and there exist two r-gfwo sets γ, ω such that $\lambda \leq$, $\mu \leq$, $\lambda\overline{q}\omega$ and $\mu\overline{q}\gamma$, then λ and μ are r-fuzzy GW -separated.

(4) If λ, μ are either both r-gfwo or both -gfwc, then $\lambda\wedge$ $(\underline{1}-\mu)$ and $\mu\wedge$ $(\underline{1}-\lambda)$ are r-fuzzy GW -separated.

Proof. (1) and (2) are obvious.

(3) Let γ and ω be r-gfwo sets such that $\lambda \leq$, $\mu \leq \omega$, $\lambda\overline{q}\omega$ and $\mu\overline{q}\gamma$; then $\lambda \leq \underline{1}-\omega$, $\mu \leq \underline{1}-\gamma$. Hence $GWC\tau$ $(\lambda, r) \leq \underline{1}-\omega$, $(\mu, r) \leq \underline{1}-\gamma$ which ii turn imply that $GW(\lambda, r)\overline{q}\mu$ and $GWC\tau(\mu, r)\overline{q}\lambda$. Thus λ and μ are r-fuzzy GW -separated.

(4) Let λ and μ be r-gfwo. Since $\lambda\wedge(\underline{1}-\mu) \leq \underline{1}-\mu$. $GWC\tau$ $(\lambda\wedge(\underline{1}-\mu), r) \leq \underline{1}-\mu$ and hence $GWC\tau(\lambda\wedge(\underline{1}-\mu),\ r)\overline{q}\mu$. Then $GWC\tau$ $(\lambda \wedge (1-\mu), r)\ \overline{q}$ $(\mu \wedge (1-\lambda))$. Again, since $\wedge$ $(\underline{1} - \lambda) \leq 1 - \lambda$. $(\mu \wedge (1 - \lambda), r) \leq 1 - \lambda$ and hence $(\mu\wedge(\underline{1}-\lambda),\ r)\ \overline{q}\lambda$. Then $GWC\tau$ $(\mu \wedge (\underline{1} - \lambda), r)\ \overline{q}$ $(\lambda \wedge (\underline{1} - \mu))$. Thus $\lambda \wedge (\underline{1} - \mu)$ and $\mu \wedge (\underline{1} - \lambda)$ are r-fuzzy GW -separated. Similarly we can prove when λ and μ are r-gfwc.

Theorem 2.2. Let (X, τ) be fts, and $\lambda, \in I^X$, $r \in I_0$.

(1) If λ, μ are r-fuzzy WG-separated and γ, $\eta \in I^X - \{\underline{0}\}$ such that $\gamma \leq \lambda$ and $\eta \leq \mu$, then , η are also r-fuzzy WG-separated.

(2) If $\lambda\overline{q}\mu$ are either both are r-wgfo or both r-wgfc, then λ and μ are r-fuzzy WG-separated.

(3) If λ, $\mu \in I^X - \{\underline{0}\}$ and there exist two r-wgfo sets γ, ω such that $\lambda \leq$, $\mu \leq \omega$, $\lambda\overline{q}\omega$ and $\mu\overline{q}\gamma$, then λ and μ are r-fuzzy WG-separated.

(4) If λ, μ are either both r-wgfo or both r-wgfc, then $\lambda\wedge(\underline{1}-\mu)$ and $\mu\wedge(\underline{1}-\lambda)$ are r-fuzzy WG-separated.

Proof. It is similarly proved as in Theorem 2.1.

Theorem 2.3. Let (X, τ) be fts, $\lambda \in I^X - \{\underline{0}\}$ and $r \in I_0$. If λ is a r-fuzzy GW-connected set such that $\lambda \leq \mu \leq (\lambda, r)$, then μ is a also r-fuzzy GW-connected.

Proof. Suppose that μ is not r-fuzzy GW-connected. Then there exist r-fuzzy GW-separated sets ω_1 and ω_2 in X such that $\mu = \omega 1 \vee \omega 2$. Let $\gamma = \lambda \wedge \omega_1$ and $\omega = \lambda \wedge \omega_2$. Then $\lambda = \gamma \vee \omega$. Since $\gamma \leq \omega_1$ and $\omega \leq \omega_2$, by Theorem 2.1. (1), γ and ω are r-fuzzy GW-separated, contradicting the r-fuzzy GW-connectedness of λ. Thus μ is r-fuzzy GW-connected.

Theorem 2.4. Let (X, τ) be a fts, $\lambda \in I^X - \{\underline{0}\}$ and $r \in I_0$. If λ is a r-fuzzy WG-connected set such that $\lambda \leq \mu \leq (\lambda, r)$, then μ is a also r-fuzzy WG-connected.

Proof. It is similarly proved as in Theorem 2.3.

3 Generalized fuzzy weakly continuous and fuzzy weakly generalized continuous

Definition 3.1. Let: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be a mapping and $r \in I_0$. Then f is called:

(1) Generalized fuzzy weakly continuous (GFW-continuous, for short) if f^{-1} (μ) is r -gfwo for each $\mu \in$, $\tau_2(\mu) \geq r$.

(2) Fuzzy weakly generalized continuous (FWG-continuous, for short) if f^{-1} (μ) is r-gfwo for each $\mu \in I^Y$, τ_2 $(\mu) \geq r$.

(3) Generalized fuzzy weakly open (GFW -open, for short) if (λ) is r-gfwo for each $\in I^X$, $\tau_1(\lambda) \geq r$.

(4) Fuzzy weakly generalized open (FWG-open, for short) if (λ) is r-wgfo for each $\lambda \in I^X$, $\tau_1(\lambda) \geq r$.

(5) Generalized fuzzy weakly closed (GFW -closed, for short) if (λ) is r-gfwc for each $\lambda \in I^X$, $\tau_1(\underline{1} - \lambda) \geq r$.

(6) Fuzzy weakly generalized closed (FWG-closed, for short) if (λ) is r-gfwc for each $\lambda \in I^r$, $\tau_1(\underline{1} - \lambda) \geq r$.

From the above definitions it is clear that the following implications are true:

F–Continuity $\rightarrow\nleftarrow$ **Fw-Continuity** $\rightarrow\nleftarrow$ **GFW-Continuity** $\rightarrow\nleftarrow$ **FGW-Continuity** $\nrightarrow\leftarrow$ **GF–Continuity** $\leftarrow\nrightarrow$ **F-Continuity.**

Example 3.1 Let $X = \{a, b\}$ and $\mu_{1,2,}\mu_3 \in I^X$ defined as follows: $\mu 1(a) = 0.6$, $\mu_1(b) = 0.4$; $\mu_2(a) = 0.3$, $\mu_2(b) = 0.6$; $\mu_3(a) = 0.4$, $\mu_3(b) = 0.6$. Define fuzzy topologies τ_1, τ_2, τ_3, $\tau_4 : I^X \longrightarrow I$ as follows:

$$\tau_1(\lambda)=\begin{cases} 1 & \text{if } \lambda \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2} & \text{if } \lambda \in \{\underline{0.1}, \underline{0.3}\}, \\ 0 & \text{otherwise} \end{cases}$$

$$\tau_2(\lambda) = \begin{cases} 1 & \text{if } \lambda \in \{0, 1\}, \\ \frac{1}{2} & \text{if } \lambda \in \underline{0.8}, \\ 0 & \text{otherwise} \end{cases}$$

$$\tau_3(\lambda) = \begin{cases} 1 & \text{if } \lambda \in \{0, 1\}, \\ \frac{1}{2} & \text{if } \lambda \in \{\mu_1, \mu_1 \vee \mu_2\}, \\ \frac{2}{3} & \text{if } \lambda \in \{\mu_1, \mu_1 \wedge \mu_2, 1 - (\mu_1 \wedge \mu_2)\}, \\ 0 & \text{otherwise,} \end{cases}$$

$$\tau_4(\lambda) = \begin{cases} 1 & \text{if } \lambda \in \{0, 1\}, \\ \frac{1}{2} & \text{if } \lambda \in \mu_3, \\ 0 & \text{otherwise} \end{cases}$$

(1) The identity mapping: $(X, \tau 1) \longrightarrow (X, \tau 2)$ is FWG-continuous (GFW -continuous) but it is neither GF-continuous nor FW -continuous because $f^{-1}(0.8) = 0.8$ it is neither $\frac{1}{2}$-gfc nor $\frac{1}{2}$ -fwc in (X, τ_1).

(2) The identity mapping: $(X, \tau_3) \longrightarrow (X, \tau_4)$ is FWG-continuous but not GFW-continuous because $f^{-1}(\mu_3) = \mu_3$ is not $\frac{1}{2}$ -fgwc in (X, τ_3).

Theorem 3.1. Let: $(X, \tau_1) \longrightarrow (Y, \tau_2)$ be GFW-continuous mapping. The following statements hold for each $\lambda \in I^X$, $\mu \in I^Y$ and $r \in I_0$.

(1) $\mathrm{f}(GWC\tau_1(\lambda, r)) \leq C\tau_2(f(\lambda), r)$.

(2) $GWC\tau_1(f^{-1}(\mu), r) \leq f^{-1}(C\tau_2(\mu, r))$.

(3) $f^{-1}(I\tau_2(\mu, r)) \leq GWI\tau 1(f^{-1}(\mu), r)$.

Proof. (1)For each $\lambda \in \mathrm{I}^X$, $\tau_2(\underline{1}-C\tau_2(f(\lambda), r)) \geq r$. Since f is GFW-continuous then f^{-1} $(C\tau_2$ $((\lambda), r))$ is r-gfwc set of X. Since $f(\lambda) \leq C\tau_2(f(\lambda), r)$ then $\lambda \leq f^{-1}(f(\lambda)) \leq f^{-1}(C\tau_2(f(\lambda), r))$ and so $GWC\tau_1(\lambda, r) \leq f^{-1}(C\tau_2(f(\lambda), r))$. Hence $(GWC\tau_1$ $(\lambda, r)) \leq C\tau_2$ $((\lambda), r)$.

(2) For each $\mu \in \mathrm{I}^Y$, τ_2 $(\underline{1} - C\tau_2$ $(\mu, r)) \geq r$. Since f is GFW -continuous then $f^{-1}(C\tau_2(\mu, r))$ is r-gfwc set of X . Since $\mu \leq C\tau_2(\mu, r)$ then $f^{-1}(\mu) \leq f^{-1}(C\tau_2(\mu, r))$ and so $GWC\tau_1(f^{-1}(\mu), r) \leq GWC\tau_1(f^{-1}(C\tau_2(\mu, r)),r) = f^{-1}(C\tau_2(\mu, r))$.

(3) For each $\mu \in \mathrm{I}^Y$, $\tau_2(I\tau_2(\mu, r)) \geq r$. Since f is GFW-continuous, f^{-1} $(I\tau_2$ $(\mu, r))$ is r-gfwo set of X. Since $I\tau_2(\mu, r) \leq \mu$ then $f^{-1}(I\tau_2(\mu, r)) \leq f^{-1(}\mu)$ and so $f^{-1}(I\tau 2(\mu, r)) = GWI\tau 1(f{-}1(I\tau 2(\mu, r)), r) \leq GWI\tau 1(f{-}1(\mu), r)$.

Theorem 3.2. Let: $(X, \tau 1) \longrightarrow (Y, \tau 2)$ be FWG-continuous mapping. The following statements hold for each $\lambda \in I^X$, $\mu \in I^Y$ and $r \in I_0$.

(1) $(WGC\tau_1$ $(\lambda, r)) \leq C\tau_2$ $(f$ $(\lambda), r)$.

(2) $WGC\tau_1$ $(f^{-1}$ $(\mu), r) \leq f^{-1}$ $(C\tau_2$ $(\mu, r))$.

(3) f^{-1} $(I\tau_2$ $(\mu, r)) \leq WGI\tau 1$ $(f^{-1}$ $(\mu), r)$.

Proof. It is similarly proved as in Theorem 3.1.

Theorem 3.3. Let: $(X, \tau_1) \to (Y, \tau_2)$ be GFW-open mapping. The following statements hold for each $\lambda \in I^X$, $\mu \in I^Y$ and $r \in I_0$.

(1) $(I\tau_1(\lambda, r)) \le GWI\tau_2(f(\lambda), r)$.

(2) $I\tau_1(f-1(\mu), r) \le f^{-1}(GWI\tau_2(\mu, r))$.

Proof. (1) For each $\lambda \in I^X$ and $r \in I^0$, since $I\tau_1(\lambda, r) \le \lambda$ then $f(I\tau_1(\lambda, r)) \le f(\lambda)$. Since τ_1 $(I\tau_1(\lambda, r)) \ge r$ and f is GFW -open, then $f(I\tau_1(\lambda, r))$ is r-gfwo. Hence $(I\tau_1(\lambda, r)) \le GWI\tau_2(f(\lambda), r)$. (2) For all $\mu \in I^Y$ and $r \in I_0$, put $\lambda = f^{-1}$ (μ). From (1), $(I\tau_1$ $(f^{-1}(\mu), r)) \le GWI\tau_2(f(f^{-1}(\mu), r) \le GWI\tau_2(\mu, r)$. Hence $I\tau_1$ $(f^{-1}(\mu), r) \le f^{-1}(f(I\tau_1(f^{-1}(\mu), r))) \le f^{-1}(GWI\tau_2(\mu, r))$.

Theorem 3.4. Let: $(X, \tau_1) \to (Y, \tau_2)$ be FWG-open mapping. The following statements hold for each $\lambda \in I^X$, $\mu \in I^Y$ and $r \in I_0$.

(1) $(I\tau_1$ $(\lambda, r)) \le WGI\tau_2$ $(f$ $(\lambda), r)$.

(2) $I\tau_1$ $(f^{-1}$ $(\mu), r) \le f^{-1}$ $(WGI\tau_2$ $(\mu, r))$.

Proof. It is similarly proved as in Theorem 3.3.

Theorem 3.5. Let: $(X, \tau_1) \to (Y, \tau_2)$ be GFW-closed. Then for each $\lambda \in I^X$ and $r \in I_0$ we have $(C\tau_1(\lambda, r)) \ge GWC\tau_2(f(\lambda), r)$.

Proof. For each $\lambda \in I^X$ and $r \in I_0$, since $\lambda \le C\tau_1(\lambda, r)$ then $f(\lambda) \le f(C\tau_1(\lambda, r))$. Since $\tau_1(1 - C\tau_1(\lambda, r)) \ge r$ then $f(C\tau_1(\lambda, r))$ is r-gfwc set of Y . Hence $(C\tau_1$ $(\lambda, r)) \ge GWC\tau_2$ $((\lambda), r)$.

Theorem 3.6. Let: $(X, \tau_1) \to (Y, \tau_2)$ be WFG-closed. Then for each $\lambda \in I^X$ and $r \in I_0$ we have $(C\tau_1$ $(\lambda, r)) \ge WGC\tau_2$ $(f$ $(\lambda), r)$.

Proof. It is similarly proved as in Theorem 3.5.

Theorem 3.7. Let f: $(X, \tau_1) \to (Y, \tau_2)$ and g: $(Y, \tau_2) \to (Z, \tau_3)$ be mappings. Then $g \circ f$ is:

(1) GFW-continuous, if f is GFW -continuous and g is F -continuous.

(2) GFW-open, if f is F-open and g is GFW-open.

(3) GFW-closed, if f is F-closed and g is GFW-closed.

(4) FWG-continuous, if f is FWG-continuous and g is F-continuous.

(5) FWG-open, if f is F-open and g is FWG-open.

(6) FWG-closed, if f is F -closed and g is FWG-closed.

Proof.

(1) Suppose that $\mu \in I^Z$, $\tau_3(\mu) \ge r$ and $r \in I_0$. Since g is F -continuous, then τ_2 $(g^{-1}(\mu)) \ge r$, since f is GFW-continuous, then $f^{-1}(g^{-1}(\mu))$ is r-gfwo set in(X,τ_1). Thus $(g \circ f)^{-1}(\mu) = f^{-1}(g^{-1}(\mu))$ is r-gfwo and therefore $g \circ f$ is GFW-continuous.

(2) Suppose that $\lambda \in I^X$, τ_1 $(\lambda) \ge r$ and $r \in I_0$. Since f is F -open, then τ_2 $(f$ $(\lambda)) \ge r$, since g is GFW-open, then g $(f$ $(\lambda))$ is r-gfwo set in (Z, τ_3).

Thus $(g \circ f)$ $(\lambda) = g$ $(f$ $(\lambda))$ is r-gfwo and therefore $g \circ f$ is GFW -open.

(3) Suppose that $\lambda \in I^X$, τ_1 $(1 - \lambda) \ge r$ and $r \in I_0$. Since f is F -closed, then τ_2 $(1 - f$ $(\lambda)) \ge r$. Since g is GFW-closed, then $((\lambda))$ is r-gfwc set in (Z, τ_3).

Thus $(g \circ f)$ $(\lambda) = g$ $(f$ $(\lambda))$ is r-gfwc and therefore $g \circ f$ is GFW -closed.

(5) and (6) are similarly proved.

4 Generalized fuzzy weakly irresolute and fuzzy weakly generalized irresolute mappings

Definition 4.1. Let: $(X, \tau_1) \to (Y, \tau_2)$ be a mapping and $r \in I_0$. Then f is called:

(1) Generalized fuzzy weakly irresolute (GFW-irresolute, for short) if $f^{-1}(\mu)$ is r-gfwo for each $\mu \in I^Y$ is r-gfwo.

(2) Fuzzy weakly generalized irresolute (FWG-irresolute, for short) if $f-1(\mu)$ is r-wgfo for each $\mu \in I^Y$ is r-wgfo.

(3) Generalized fuzzy weakly irresolute open (GFW-irresolute open, for short) if (λ) is r-gfwo for each $\lambda \in I^X$ is r-gfwo.

(4) Fuzzy weakly generalized irresolute open (FWG-irresolute open, for short) if (λ) is r-wgfo for each $\lambda \in I^X$ is r-wgfo.

(5) Generalized fuzzy weakly irresolute closed (GFW -irresolute closed, for short) if (λ) is r-gfwc for each $\lambda \in I^X$ is r-gfwc.

(6) Fuzzy weakly generalized irresolute closed (FWG-irresolute closed, for short) if (λ) is r-gfwc for each $\lambda \in I^X$ is r-wgfc.

(7) GFW-irresolute homeomorphism iff is bijective and both of f and f^{-1} are GFW -irresolute.

(8) FWG-irresolute homeomorphism iff is bijective and both of f and f^{-1} are FWG-irresolute.

From the above definitions it is clear that the following implications are true:

GFW-irresolute →↚ GFW-continuity →↚ FWG–continuity irresolute →↚ FWG - irresolute

Example 4.1 Let $X = \{a, b\}$ and $\mu_1, \mu_2, \mu_3 \in I^X$ defined as follows: $\mu_1(a) = 0.6$, $\mu(b) = 0.4$; $\mu_2(a) = 0.3$, $\mu_2(b) = 0.6$; $\mu_3(a) = 0.4$, $\mu_3(b) = 0.6$. Define fuzzy topologies
$\tau_1, \tau_2: I^X \longrightarrow I$ as follows:

$$\tau_1(\lambda) = \begin{cases} 1 & \lambda \in \{0,1\}, \\ \frac{1}{2} & \text{if } \lambda \in \{\mu_1, \mu_1 \vee \mu_2\}, \\ \frac{2}{3} & \mu_2, 1 - (\mu_1 \wedge \mu_2)\}, \\ 0 & \text{otherwise,} \end{cases}$$

$$\tau_2(\lambda) = \begin{cases} 1 & \text{if } \lambda \in \{0,1\}, \\ \frac{1}{2} & \text{if } \lambda \in 0.1, \\ 0 & \text{otherwise} \end{cases}$$

The identity mapping: $(X, \tau_1) \longrightarrow (X, \tau_2)$ is GFW-continuous but not GFW irresolute because $f^{-1}(\mu_3) = \mu_3$ is not $\frac{1}{2}$-gfwc in (X, τ_1).

Example 4.2 Let $X = \{a, b, c\}$ and $\mu_1, \mu_2, \mu_3\ \mu_4, \in I^X$, defined as follows: $\mu_1 = \underline{0.6}$; $\mu_2 = \underline{0.4}$; $\mu_3 = \underline{0.1}$; $\mu_3(b) = \underline{0.1}$, $\mu_3(c) = \underline{0.1}$, $\mu_4(a) = \underline{0.5}$, $\mu_4(b) = \underline{0.6}$, $\mu_4(c) = \underline{0.5}$. Define fuzzy topologies $\tau_1, \tau_2: I^X \longrightarrow I$ as follows:

$$\tau_3(\lambda) = \begin{cases} 1 & \text{if } \lambda \in \{0,1\}, \\ \frac{1}{2} & \text{if } \lambda \in \{\mu_1, \mu_2\}, \\ 0 & \text{otherwise} \end{cases}$$

$$, \quad \tau_2(\lambda) = \begin{cases} 1 & \text{if } \lambda \in \{0,1\}, \\ \frac{1}{2} & \text{if } \lambda \in \mu_1, \\ 0 & \text{otherwise} \end{cases}$$

The identity mapping: $(X, \tau_1) \longrightarrow (X, \tau_2)$ is FWG-continuous but not FWG irresolute because $f^{-1}(\mu_4) = \mu_4$ is not $\frac{1}{2}$-gfwc in (X, τ_1).

Theorem 4.1. Let: $(X, \tau_1) \longrightarrow (Y, \tau 2)$ be GFW -irresolute mapping. The following statements hold for each $\lambda \in I^X$, $\mu \in I^Y$ and $r \in I_0$.

(1) $(GWC\tau_1(\lambda, r)) \le WC\tau_2(f(\lambda), r)$.

(2) $GWC\tau_1(f^{-1}(\mu), r) \le f^{-1}(WC\tau_2(\mu, r))$.

(3) $f^{-1}(WI\tau_2(\mu, r)) \le GWI\tau_1(f^{-1}(\mu), r)$.

(4) If f is bijective, then $WI\tau_2((\lambda), r) \le (GWI\tau_1(\lambda, r))$.

Proof. For each $\lambda \in I^X$, $WC\tau_2(f(\lambda), r))$ is r-gfwc set of . Since f is GFW-irresolute then $f^{-1}(WC\tau_2(f(\lambda), r))$ is r-gfwc set of X. Since $f(\lambda) \le WC\tau_2(f(\lambda), r)$ then $\lambda \le f^{-1}(f(\lambda)) \le f^{-1}(WC\tau_2(f(\lambda), r))$ and $GWC\tau_1(\lambda, r) \le f^{-1}(WC\tau_2(f(\lambda), r))$. Hence $(GWC\tau_1(\lambda, r)) \le WC\tau_2((\lambda), r)$.

(2) For each $\mu \in I^Y$, $WC\tau_2(\mu, r)$ is r-gfwc set of Y. Since f is GFW-irresolute then $f^{-1}(WC\tau_2(\mu, r))$ is r-gfwc set of X . Since $\mu \le WC\tau_2(\mu, r)$ then $f^{-1}(\mu) \le f^{-1}(WC\tau_2(\mu, r))$. Then $GWC\tau_1(f^{-1}(\mu), r) \le GWC\tau_1(f^{-1}(WC\tau_2(\mu, r)), r) = f^{-1}(WC\tau_2(\mu, r))$.

(3) For each $\mu \in I^Y$, $WI\tau_2(\mu, r))$ is r-gfwo set of Y . Since f is GFW-irresolute, then $f^{-1}(WI\tau_2(\mu, r))$ is r-gfwo set of X . Since $WI\tau_2(\mu, r) \le \mu$ then $f^{-1}(WI\tau_2(\mu, r)) \le f^{-1}(\mu)$ and so $f^{-1}(WI\tau_2(\mu, r)) = GWI\tau_1(f^{-1}(WI\tau_2(\mu, r)), r) \le GWI\tau_1(f^{-1(}\mu), r)$.

(4) Let f be GFW -irresolute and $\lambda \in I^X$, $r \in I_0$. Then $f^{-1}(WI\tau_2(f(\lambda), r))$ is r-gfwo. By (3), and the fact that f is injective, we have$f^{-1}(WI\tau_2(f(\lambda),r)) \le GWI\tau_1(f^{-1}(f(\lambda)), r) = GWI\tau_1(\lambda, r)$ since f is surjective, we have $WI\tau_2(f(\lambda), r)) = (f(f^{-1}(GWI\tau_1(\lambda, r))) \le f(GWI\tau_1(\lambda, r))$.

Theorem 4.2. Let: $(X, \tau_1) \to (Y, \tau_2)$ be FWG-irresolute mapping. The following statements hold for each $\lambda \in I^X$, $\mu \in I^Y$ and $r \in I_0$.

(1) $(WGC\tau_1(\lambda, r)) \le WC\tau_2(f(\lambda), r)$.

(2) $WGC\tau_1(f^{-1}(\mu), r) \le f^{-1}(WC\tau_2(\mu, r))$.

(3) $f^{-1}(WI\tau_2(\mu, r)) \le WGI\tau_1(f^{-1}(\mu), r)$.

(4) If f is bijective, then $WI\tau_2(f(\lambda), r) \le f(WGI\tau_1(\lambda, r))$.

Proof. It is similarly proved as in Theorem 4.1.

Theorem 4.3. Let: $(X, \tau_1) \to (Y, \tau_2)$ be GFW-irresolute open mapping. The following statements hold for each $\lambda \in I^X$, $\mu \in I^Y$ and $r \in I_0$.

(1) $(WI\tau_1(\lambda, r)) \le GWI\tau_2(f(\lambda), r)$.

(2) $WI\tau_1(f^{-1}(\mu), r) \le f^{-1}(GWI\tau_2(\mu, r))$.

Proof. (1) For each $\lambda \in IX$ and $r \in I0$, since $WI\tau1(\lambda, r) \le \lambda$ then $f(WI\tau1(\lambda, r)) \le f(\lambda)$. Since $WI\tau1(\lambda, r)$ is r-gfwo and f is GFW -irresolute open, then $f(WI\tau1(\lambda, r))$ is r-gfwo. Hence $(WI\tau1(\lambda, r)) \le GWI\tau2(f(\lambda), r)$. (2) For all $\mu \in IY$ and $r \in I0$, put $\lambda = f-1(\mu)$. From (1), $(WI\tau1(f-1(\mu), r)) \le GWI\tau2(f(f-1(\mu), r) \le GWI\tau2(\mu, r)$. Then $WI\tau1(f-1(\mu), r) \le f-1(f(WI\tau1(f-1(\mu), r))) \le f-1(GWI\tau2(\mu, r))$.

Theorem 4.4. Let: $(X, \tau_1) \to (Y, \tau_2)$ be FWG-irresolute open mapping. Then following statements hold for each $\lambda \in I^X$, $\mu \in I^Y$ and $r \in I_0$.

(1) $(WI\tau_1(\lambda, r)) \le WGI\tau_2(f(\lambda), r)$.

(2) $WI\tau_1(f-1(\mu), r) \le f-1(WGI\tau_2(\mu, r))$.

Proof. It is similarly proved as in Theorem 4.3.

Theorem 4.5. Let: $(X, \tau_1) \to (Y, \tau_2)$ be GFW-irresolute closed mapping and bijective. The following statements hold for each $\lambda \in I^X$, $\mu \in I^Y$ and $r \in I_0$.

(1) $(WC\tau_1(\lambda, r)) \ge GWC\tau_2(f(\lambda), r)$.

(2) $f^{-1}(GWC\tau_2(\mu, r)) \le WC\tau_1(f^{-1}(\mu), r)$.

(3) f is GFW-irresolute closed iff f is GFW-irresolute open.

Proof. (1) For each $\lambda \in I^X$ and $r \in I_0$, since $\lambda \le WC\tau_1(\lambda, r)$ then $f(\lambda) \le f(WC\tau_1(\lambda, r))$. Since $WC\tau_1(\lambda, r)$ is r-gfwc set of X , then $f(WC\tau_1(\lambda, r))$ is gfwc set of Y . Hence $(WC\tau_1(\lambda, r)) \ge GWC\tau_2(f(\lambda), r)$.

(2) Let f be GFW-irresolute closed. By (1), then For each $\lambda \in I^X$ and $r \in I_0$, we have $(WC\tau_1(\lambda, r)) \ge GWC\tau_2(f(\lambda), r)$. For all $\mu \in I^Y$ and $r \in I_0$. Put $\lambda = f^{-1}(\mu)$. Since f is surjective, $(f^{-1}(\mu)) = \mu$. Thus, $(WC\tau_1(f^{-1}(\mu), r)) \ge GWC\tau_2(f(f^{-1}(\mu), r) = GWC\tau_2(\mu, r)$. It implies $WC\tau_1(f^{-1}(\mu), r) = f^{-1}(f(WC\tau_1(f^{-1}(\mu), r))) \ge f^{-1}(GWC\tau_2(\mu, r))$ (by f is injective).

(3) It is easily proved.

Theorem 4.6. Let: $(X, \tau_1) \to (Y, \tau_2)$ be FWG-irresolute closed mapping and bijective. The following statements hold for each $\lambda \in IX$, $\mu \in IY$ and $r \in I_0$.

(1) $(WC\tau_1(\lambda, r)) \geq WGC\tau_2(f(\lambda), r)$.

(2) $f^{-1}(WGC\tau_2(\mu, r)) \leq WC\tau_1(f^{-1}(\mu), r)$.

(3) f is FWG-irresolute closed iff f is FWG-irresolute open.

Proof. It is similarly proved as in Theorem 4.5.

Theorem 4.7. If: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be F -irresolute, F -open and bijective map. Then f is FWG-irresolute.

Proof. Let $\mu \in I^Y$ be a r-wgfc set. We will show that $f^{-1}(\mu)$ is r-wgfc set. Let $f^{-1}(\mu) \leq \gamma$ with $\tau_1(\gamma) \geq r$. Since f is onto and - open, $\mu = (f^{-1}(\mu)) \leq f(\gamma)$ with $\tau_2(f(\gamma)) \geq r$. Since μ is r-wgfc, $WC\tau_2(\mu, r) \leq f(\gamma)$. Since f is injective, $f^{-1}(WC\tau_2(\mu, r)) \leq f^{-1}(f(\gamma)) = \gamma$. Since f is F -irresolute, $f^{-1}(WC\tau_2(\mu, r))$ is r-wgfc. Hence, $WC\tau_1(f^{-1}(\mu), r)) \leq WC\tau_1(f^{-1}(WC\tau_2(\mu,r), r)), r) \leq \gamma$. Thus, $f^{-1}(\mu)$ is r-wgfc set.

Theorem 4.8. If: $(X, \tau_1) \rightarrow (Y, \tau_2)$ and: $(Y, \tau_2) \rightarrow (Z, \tau_3)$ be mappings and $r \in I_0$. Then $g \circ f$ is:

(1) GFW-irresolute (resp. FWG-irresolute), if f and g are both GFW irresolute (resp. FWG-irresolute).

(2) GFW-irresolute open (resp. FWG-irresolute open), if f and g are both GFW-irresolute open (resp. FWG-irresolute open).

(3) GFW-irresolute closed (resp. FWG-irresolute closed), if f and g are both GFW-irresolute closed (resp. FWG-irresolute closed).

(4) GFW-continuous (resp. FWG-continuous), if f is GFW-irresolute (resp. FWG-irresolute) and g is GFW-continuous (resp. FWG-continuous).

(5) GFW-open (resp. FWG-open), if f is GFW-open (resp. FWG-open) and g is GFW -irresolute open (resp. FWG-irresolute open).

(6) GFW-closed (resp. FWG-closed), if f is GFW-closed (resp. FWGclosed) and g is GFW-irresolute closed (resp. FWG-irresolute closed).

Proof. It is similarly proved as in Theorem 3.7.

Definition 4.2. A fts (X, τ) is called r-$WFT1/2$ if $(1-\lambda) \geq r$ for each r-wgfc set $\lambda \in IX$ and $r \in I_0$.

It is clear that r-$WFT1/2$ implies that r-$FT_{1/2}$.

Theorem 4.9. A fts (X,τ) is called r-$WFT_{1/2}$ iff $W(\lambda,r) = C\tau(\lambda,r)$ for each $\lambda \in I^X$ and $r \in I_0$.

Proof. (⇒) Let (X, τ) be r-$WFT_{1/2.}$ By definition of $WGC\tau$ and $C\tau$, we have $(\lambda,) = C\tau(\lambda,r)$ for each $\lambda \in I^X$ and $r \in I_0$. (⇐) Suppose (X,τ) is not r-$WFT_{1/2}$. There exist r-wgfc $\mu \in I^X$ and $r \in I_0$ such that $\tau(1 - \lambda) < r$. Hence $WGC\tau$ $(\mu, r) = \mu$ but $C\tau$ $(\mu,\tau) \neq \mu$. Thus $WGC\tau$ $(\lambda, \tau) \neq C\tau(\lambda,r)$ it is a contradiction. Then (X,τ) is r-$WFT_{1/2}$.

Theorem 4.10. Let: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be mappings and $r \in I_0$. Then following statements hold.

(1) If (X,τ_1) is r-$WFT_{1/2}$, then the concepts of F-continuity, WF-continuity, GFW-continuity, FWG-continuity, and GF-continuity are equivalent.

(2) If (Y,τ_2) is r-$WFT_{1/2}$, then the concepts of GFW-continuity and GFW irresolute are equivalent. Also, concepts of FWG-continuity and FWGirresolute are equivalent.

(3) If $(X,_1)$ and (Y,τ_2) are r-$WFT_{1/2}$, then the concepts of F continuity, FW-continuity, GFW -continuity, FWG-continuity, GF continuity,

GFW-irresolute and FWG-irresolute are equivalent.

Theorem 4.11. Let f: $(X, \tau_1) \rightarrow (Y, \tau_2)$ and g: $(Y, \tau_2) \rightarrow (Z, \tau_3)$ be GFW-continuous (resp. FWG-continuous) and (Y, τ_2) be r-$WFT_{1/2}$. Then $g \circ$ f : $(X, \tau_1) \rightarrow (Z, \tau_3)$ is GFW-continuous (resp. FWG-continuous).

5 Some applications of r-weakly generalized fuzzy closed sets

Definition 5.1. A fts (X, τ) is said to be:

(1) r-FWG-regular iff $x_t\overline{q}\lambda$ for each r-wgfc $\lambda \in I^X$ implies that there exists $\mu_i \in I^X$ with $(\mu_i) \geq r$ for $i \in \{1, 2\}$ such that $x_t \in \mu_1$, $\lambda \leq \mu_2$ and $\mu_1\overline{q}\mu_2$.

(2) r-FWG-normal iff $\lambda 1\overline{q}\lambda 2$ for each r-wgfc sets $\lambda i \in I^X$ for $i \in \{1, 2\}$ implies that there exists $\mu_i \in I^X$ with $\tau\ (\mu i) \geq r$ for such that $\lambda i \leq \mu i$ and $\mu_1\overline{q}\mu_2$.

Theorem 5.1. Let (X, τ) be a fts and $r \in I_{0.}$ Then the following statements are equivalent.

(1) (X, τ) is r-FWG-regular.

(2) If $x_t \in \lambda$ for each r-wgfc $\lambda \in \mathrm{I^X}$, there exists $\mu \in I^X$ with $(\mu) \geq r$ such that $x_t \in \mu \leq (\mu,) \leq \lambda$.

(3) If $x_t\overline{q}\lambda$ for each r-wgfc $\lambda \in \mathrm{I^X}$, there exists $\mu i \in I^X$ with $\tau\ (\mu_i) \geq r$ for $i \in \{1, 2\}$ such that $x_t \in \mu_1$, $\lambda \leq \mu_2$ and $(\mu_1,)\ \overline{q}C\tau(\mu_2,r)$.

Proof. (1) ⇒ (2) Let $x_t \in \lambda$ for each r-wgfo λ. Then $(\underline{1} - \lambda)$ for $r$wgfc $(\underline{1} - \lambda)$. Since (X, τ) is r-FWG-regular, there exist $\mu, \gamma \in I^X$ with $(\mu) \geq r$, $(\gamma) \geq r$ such that $xt \in \mu$, $\underline{1} - \lambda \leq \gamma$ and μ. It implies $x_t \in \mu \leq 1 - \gamma \leq \lambda$. Since $(\gamma) \geq r$, $x_t \in \mu \leq (\mu,) \leq \lambda$.

(2) ⇒ (3) Let $xtq\lambda$ for each r-wgfc. Then $x_t \in \underline{1} - \lambda$ for r-wgfo $\underline{1} - \lambda$. By (2), there exist $\mu \in I^X$ with $\tau(\mu) \geq r$ such that $x_t \in \mu \leq (\mu, \mathrm{r}) \leq \underline{1} - \lambda$. Since $\tau(\mu) \geq r$, μ is r-wgfo and $x_t \in \mu$. Again, by (2), there exist $\mu_1 \in I^X$ with $(\mu_1) \geq r$ such that $x_t \in \mu_1 \leq (\mu_1,\mathrm{r}) \leq \mu \leq C\tau(\mu,r) \leq \underline{1} - \lambda$. It implies $\lambda \leq (\underline{1}-(\mu,r)) = I\tau((\underline{1}-\mu),r) \leq \underline{1}-\mu$. Put $\mu_2 = \mathrm{I_r}\ (\underline{1}-\mu, \mathrm{r})$, then $\tau(\mu_2) \geq r$. So, $\mathrm{C_\tau}(\mu 2,\mathrm{r}) \leq \underline{1} - \mu \leq \underline{1} - C(\mu_1,r)$, that is, $C\tau(\mu 1,r)\overline{q}C\tau(\mu 2,r)$. (3) ⇒ (1) It is trivial.

Theorem 5.2. Let (X, τ) be a fts and $r \in I_0$. Then the following statements are equivalent.

(1) (X, τ) is r-FWG-normal.

(2) If $\gamma \leq \lambda$ for each r-wgfc set $\gamma \in I^X$ and r-wgfo set $\lambda \in I^X$, there exists $\mu \in I^X$ with $\tau(\mu) \geq r$ such that $\gamma \leq \mu \leq C\tau(\mu,r) \leq \lambda$.

(3) If $\lambda 1\overline{q}\lambda 2$ for each r-wgfc sets $\lambda i \in I^X$ for $i \in \{1, 2\}$, there exists $\mu i \in IX$ with τ $(\mu i) \geq r$ such that $\lambda i \leq \mu i$ and $(\mu_1,)\overline{q}C\tau(\mu_2,r)$.

Proof. It is similarly proved as in Theorem 5.1.

Theorem 5.3. Let (X, τ) be a fts. Then the following statements hold.

(1) Every r-FWG-regular (r-FWG-normal) space is r-GFR_2 (r-GFR_3).

(2) Every r-FWG-regular (r-FWG-normal) space is r-FR_2 (r-FR_3).

(3) A fts (X, τ) is r-FWG-regular (r-FWG-normal) iff it is r-GFR_2 ($rGFR_3$) and r-$FWT_{1/2}$.

(4) A fts (X, τ) is r-FWG-regular (r-FWG-normal) iff it is r-$FR2$ (r-FR_3) and r-$FWT_{1/2.}$

Proof. (1) For $x_t\overline{q}\lambda$ with r-gfc set $\lambda \in I^X$, then λ is r-wgfc. Since (X, τ) is r-FWG-regular, there exist $\mu i \in I^X$ with $\tau(\mu i) \geq r$ for $i \in \{1, 2\}$ such that $xt \in \mu_1$, $\lambda \leq \mu_2$ and $\mu_1 q \mu_2$. Hence (X, τ) is r-GFR_2.

(2) For $x_t\overline{q}\lambda$ with $\tau(\underline{1} - \lambda) \geq r$, then λ is r-wgfc. Since (X, τ) is r-FWGregular, there exist $\mu i \in I^X$ with $\tau(\mu i) \geq r$ for $i \in \{1,2\}$ such that $x_t \in \mu_1$, $\lambda \leq \mu_2$ and $\mu_1\overline{q}\mu_2$. Hence (X, τ) is r-FR_2.

(3) (⇒) Let (X, τ) be r-FWG-regular. By (1), we only show that (X, τ) is r-$FWT_{1/2}$. If $\lambda \in \{0, 1\}$, then λ is r-wgfc and $\tau(\lambda) = 1$. Let $\lambda \notin \{0, 1\}$ be r-wgfc. For $x_t \in (\underline{1} - \lambda)$ with r-wgfc λ, by Theorem 5.1.(2), There exists $\mu x_t \in I^X$ with $\tau(\mu x_t) \geq r$ such that $xt \in \mu x_t \leq C\tau(\mu x_t,r) \leq \underline{1} - \lambda$. Hence $\underline{1} - \lambda = \vee\{\mu x_t | C\tau(\mu x_t,r) \leq \underline{1} - \lambda, \tau(\mu x_t) \geq r\}$. So, $\tau(1 - \lambda) \geq r$. Hence (X, τ) is r-$FWT_{1/2.}$ (⇐) It is easily proved.

(4) (⇒) It is easily proved from (2) and (3).

(⇐) It is easily proved.

Theorem 5.4. Let (X, τ) be a fts. Then the following statements hold.

(1) If (X, τ) is r-FWG-regular, then it is r-FR_0.

(2) If (X, τ) is r-FWG-regular, then it is r-$FT2_{1/2.}$

(3) If (X, τ) is r-FWG-regular, then it is r-FT_3.

(4) If (X, τ) is r-FWG-normal and r-FR_0, then it is r-FWG-regular.

(5) If (X, τ) is r-FWG-normal and r-FR_0, then it is r-FT_4.

Proof. (1) Let $x_t\overline{q}(y_s,r)$ for any distinct fuzzy points $x_t,y_s \in Pt(X)$. Since $C_\tau(ys,r)$ is r-wgfc and (X, τ) is r-FWG-regular, then there exist $\mu_i \in I^X$ with $\tau(\mu i) \geq r$ for $i \in \{1,2\}$ such that $x_t \in \mu_1$, $y_s \in C\tau(y_s,r) \leq \mu_2$ and $\mu_1\overline{q}\mu_2$. It implies $x_t \in \mu_1 \leq 1 - \mu_2 \leq 1 - (y_s,r) \leq \underline{1} - y$. Thus, $(x_t,r) \leq \underline{1} - y$, that is, $y_s\overline{q}C\tau(x_t,r)$. Hence (X, τ) is r-FR_0.

(2) Let $x_t\overline{q}y_s$ for any distinct fuzzy points x_t,y_s ∈ Pt(X). Since (X, τ) is r-FWG-regular, by (1) and Theorem 1.3. (iii), y_s is r-wgfc. By Theorem 5.1. (3), then there exist $\mu i \in I^X$ with $(\mu_i) \geq r$ for $i \in \{1,2\}$ such that, $x_t \in \mu_1$, $y_s \in \mu_2$ and $C\tau(\mu_1,r)qC\tau(\mu_2,r)$. Hence (X, τ) is r-$FT_{21/2}$.

(3) Let (X, τ) is r-FWG-regular. By (2) and Theorem 5.3.(2), (X, τ) is r-$FT2_{1/2}$ and r-FR_2 . Since r-$FT_{21/2}$ implies r-FT_1, (X, τ) is r-FT_3.

(4) Let $x\overline{tq}\lambda$ for each r-wgfc λ. Since (X, τ) is r-FR_0, then x_t is r-wgfc. Since (X, τ) is r-FWG-normal, then there exist $\mu_i \in I^X$ with $(\mu_i) \geq r$ for $i \in \{1, 2\}$ such that $x_t \in \mu_1$, $\lambda \in \mu_2$ and $\mu_1\overline{q}\mu_2$. Hence (X, τ) is -FWG-regular.

(5) Let (X, τ) is r-FWG-normal and r-FR_0. Since r-FWG-regular implies r-$FT_{21/2}$ implies r-FT_1 , by (4), (X, τ) is r-FT_1 . By Theorem 5.3.(2), (X, τ) is r-FT_4 .

Theorem 5.5. If: $(X, \tau_1) \to (Y, \tau_2)$ be F -irresolute, F -open and bijective map and (X, τ_1) is r-FWG-regular (resp. r-FWG-normal), then (Y, τ_2) is r-FWG-regular (resp. r-FWG-normal).

Proof. Let $y_s\overline{q}\mu$ for each r-wgfc $\mu \in I^Y$. Since f is F -irresolute, F open and bijective map, then by Theorem 4.7., f is FWG-irresolute. Hence $f^{-1}(\mu)$ is r-wgfc set. Put y_s = f (xs). Then $x_s\overline{q}f^{-1}(\mu)$. Since (X, τ) is r-FWG-regular, there exist $\mu i \in I^X$ with $(\mu_i) \geq r$ for $i \in \{1, 2\}$ such that $xs \in \mu_1$, $f^{-1}(\mu) \leq \mu_2$ and $\mu_1\overline{q}\mu_2$. Since f is F -open and bijective map, we have $y_s \in \tau(\mu_1)$, $\mu = (f^{-1}(\mu)) \leq f(\mu 2)$, $f(\mu 1)qf(\mu 2)$. Hence (Y, τ_2) is r-FWGregular. Other case is similarly proved.

Theorem 5.6. If: $(X, \tau_1) \to (Y, \tau_2)$ be F-continuous, FWG-irresolute closed and injective map and (Y, τ_2) is r-FWG-regular (resp. r-FWG-normal), then (X, τ_1) is r-FWG-regular (resp. r-FWG-normal).

Proof. Let $x_t\overline{q}\lambda$ for each r-wgfc set $\lambda \in I^X$. Since f is FWG-irresolute closed, f(λ) is r-wgfc. Since f is injective, $x_t\overline{q}\lambda$ implies f$(x_t)\,\overline{q}$ f(λ). Since (Y, τ_2) is r-FWG-regular, there exist $\mu i \in I^X$ with $(\mu_i) \geq r$ for $i \in \{1, 2\}$ such that $(x_t) \in \mu_1$, $(\lambda) \leq \mu_2$ and $\mu_1\overline{q}\mu_2$. Since f is F -continuous,

$x_t \in f^{-1}$ (μ_1), $\lambda \leq f^{-1}$ (μ_2), with $\tau_2(f^{-1}(\mu_i)) \geq r$ and $i \in \{1,2\}$ and $f^{-1}(\mu_1)\overline{q}f^{-1}(\mu_2)$. Hence (X, τ_1) is r-FWG-regular. Other case is similarly proved.

Theorem 5.7. If: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be FWG-irresolute, F -open, F -closed and surjective map and (X, τ_1) is r-FWG-regular (resp. r-FWG-normal), then $(Y, \tau 2)$ is r-FWG-regular (resp. r-FWG-normal).

Proof. Let $y_s \in \mu$ for each r-wgfc $\mu \in I^Y$. Since f is FWG-irresolute and surjective, then there exist $x \in f^{-1}$ $(\{y\})$ such that $x_s \in f^{-1}$ (μ) with $r$wgfo set $f-1$ (μ). Since (X, τ_1) is r-FWG-regular by Theorem 5.1. (2), there exist $\gamma \in I^X$ with $\tau_1(\gamma) \geq r$ such that $x_s \in \gamma \leq C\tau_1(\gamma,r) \leq f^{-1}(\mu)$. It implies $y_s \in (\gamma) \leq (C\tau_1\ (\gamma, r)) \leq \mu$. Since f is F -open and F -closed, then $\tau_2\ (f\ (\gamma)) \geq r$ and $\tau_2\ (1 - f\ (C_{\tau 1}\ (\gamma, r))) \geq r$. Hence $y_s \in f(\gamma) \leq C\tau_2(f(\gamma),r) \leq C\tau_2(f(C\tau_1(f(\gamma,r)),r) \leq \mu$. Thus, $(Y, \tau 2)$ is r-FWG-regular. Other case is similarly proved.

References

[1] S. E. Abbas, Fuzzy S -irresolute mappings, Appl. Math. Compu., 155(2) (2004), 329-343.

[2] K. Balachandran, P. Sundaram and H. Maki, On generalized continuous maps in topological spaces, Mem. Fac. Sci. Kochi. Univ., 12 (1991), 5-13.

[3] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl., 24 (1968), 182-190.

[4] K. C. Chattopadhyay, R. N. Hazra and S. K. Samanta, Gradation of openness: Fuzzy topology, Fuzzy Sets and Systems, 49 (1992), 237-242.

[5] K. C. Chattopadhyay and S. K. Samanta, Fuzzy topology, Fuzzy closure operator, Fuzzy compactness and Fuzzy connectedness, Fuzzy Sets and Systems, 54(2) (1993), 207-212.

[6] W. Dunham and N. Levine, Further results on generalized closd set in topology, Kyungpook Math. J., 20 (1980), 169-175.

[7] W. Dunham, A new closure operator for non-Ti topologies, Kyungpook Math. J., 22 (1982), 55-60.

[8] M. E. El-Shafei and A. Zakari, Semi-generalized continuous mapping in fuzzy topological spaces, J. Egy. Math. Soc., 15(1) (2007), 57-67.

[9] N. Gowrisankar and N. Rajesh, New separation axioms in smooth fuzzy topology, The Journal of Fuzzy Mathematics, 21(1) (2013), 29-38.

[10] U. H ¨ $o$hle, Upper semicontinuous fuzzy sets and applications, J. Math. Anal. Appl., 78 (1980), 659-673.

[11] U. H ¨ $o$hle and A. P. ˇ S ostak, A general theory of fuzzy topological spaces, Fuzzy Sets and Systems, 73 (1995), 131-149.

[12] U. H ¨ $o$hle and A. P. ˇ S ostak, Axiomatic foundations of fixed-basis fuzzy topology, The Hand-books of fuzzy sets series, Volume 3, Kluwer academic publishers, Dordrecht (Chapter 3) (1999).

[13] A. Kandil and M. E. El-Shafee, Regularity axioms in fuzzy topological spaces and FRi -proximities, Fuzzy Sets and Systems, 27 (1988), 217-231.

[14] Y. C. Kim, Separation axioms in smooth fuzzy topological spaces, J. of Korea Fuzzy Logic and Intelligent Systems, 9(1) (1991), 57-62.

[15] Y. C. Kim and J. W. Park, Some properties of r-generalized fuzzy closed sets, Far East J. of Math. Science, 7(3) (2002), 253-268.

[16] Y. C. Kim, A. A. Ramadam and S. E. Abbas, Weaker forms of continuity in ˇ S ostak, fuzzy topology, Indian J. Pure Appl. Math., 34(2) (2003), 311-333.

[17] Y. C. Kim and J. M. Ko, r-generalized fuzzy closed sets, J. Fuzzy Math., 12 (1) (2004), 7-21.

[18] J. Mahanta and P. K. Das, On fuzzy weakly closed sets, In press.

[19] C. K. Park and W. K. Min, r-generalized fuzzy compactness, J. Korea Soc. Math. Educ. Ser. B Pure Appl. Math., 14 (2007), 255-270.

[20] A. A. Ramadan, Smooth topological spaces, Fuzzy Sets and Systems, 48 (1992), 371-375.

[21] A. A. Ramadan, S. E. Abbas and Y. C. Kim, Fuzzy irresolute mappings in smooth fuzzy topological spaces, J. Fuzzy Math., 9(4) (2001), 865-877. [24] A. P. ˇ *S* ostak, On a fuzzy topological structure, Suppl. Rend. Circ. Matem. Palerms, ser II , 11 (1985), 89-103. [25] A. P. ˇ *S* ostak, Two decades of fuzzy topology: Basic ideas, Notion and results, Russian Math. Surveys, 44(6) (1989), 125-186. [26] A. P. ˇ *S*ostak, Basic structure of fuzzy topology, J. Math. Sci., 78(6) (1996), 662-701.