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Mathematical Modeling of Half Car and Full Car Model to Study the Biodynamic Response of Tractor Occupant Subjected to Whole Body Vibration

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Abstract— **Persistent exposure of humans to whole body vibration can be experienced in our daily work activities. These vibrations can result in motion sickness, discomfort and even severe health effects .This project work aims to study and analyze human vibration transmitted to a tractor driver by carrying out analytical work to measure Whole Body Vibration (WBV) and thus make them aware about the vibration limits within which their health will be safe. A half car model of tractor occupant system subjected to periodic excitation was modelled and its equations of motions were arrived. The model was then solved using Mathematica Systemmodler software and thereby the acceleration versus time plot for each mass is obtained. Also the half car model is modified with full car model considering pitch and roll vibration in addition to vertical vibration and thereby enhancing the accuracy of the result obtained from mathematical model. The optimum values of suspension parameters are to be found out using genetic algorithm technique.**

Keywords— Whole body Vibration, Half car model, full car model, motion sickness, road excitation

I. INTRODUCTION (*HEADING 1*)

Their ability to perform complex tasks under adverse vibrational conditions formed part of the first investigations [1]. Human exposure to vibration can be broadly classified as hand arm and whole body vibration. Whole body vibration may be transmitted through supporting surfaces such as feet, buttocks and back as in case of vehicle drivers whereas hand arm vibration is transmitted through human arms as in case of operators of hand tools such as jack hammer, portable drilling machine. Exposure to whole-body vibration (WBV), particularly to large shocks and jolts, is a back-pain health risk for employees who drive mobile machines or other work vehicles over poor surfaces as a main part of their job. All on-road and off-road vehicles expose their drivers to some kind of vibration. Vehicles which are driven on well maintained public roads such as vans, lorries and buses, may also expose their drivers to some WBV, but the levels are likely to be relatively low and therefore the likelihood of related health risks is low. On the other hand vehicles which are driven on rough terrain conditions such as tractors, ploughers. All-terrain vehicles (ATV's) the levels of exposure are likely to be slightly more critical. The extended period of sitting on such off-road vehicles include a higher risk of back problem, numbness and discomfort in the buttocks due to surface pressure and discomfort in the legs and feet from pressure under the thighs [2]. Experimental studies on transmission and tolerance of vertical vibrations, caused by tractors indicate that man is exposed to vibrations where he is vibrated (in the frequency range 0.5-l 1 Hz) at levels of intensity above the so-called uncomfortable or unpleasant thresholds and is frequently exposed to vibration intensities above even the so-called intolerable and extremely uncomfortable levels [3]. Tractor ride comfort depends on three co-ordinates namely magnitude of response, frequency of response and duration of exposure [4]. Measurements of vibration levels at the seats of various tractor models have shown that the risk to the tractor driver's health exists, even in case of drivers who are exposed to vibrations only one hour a day, while the risk is probable for all others with longer periods of exposure. Health disorders tend to show gradually, usually after two to seven years at the workplaces where operators are exposed to these vibrations. Daily exposure to whole-body vibration over a number of years can result in serious physical damage, for example, ischemic lumbago.

This paper addresses to provide a method for determining ride comfort by developing a half car mathematical model of tractor occupant system. The mathematical results are to be checked with the limits specified by ISO 2631 [5, 6, and 7]. The accuracy of the analytical results can be enhanced by developing a full car model which considers pitch and roll vibrations which are quite predominant in agriculture vehicles.

Nomenclature

- m_h, C_h, k_h -mass, damping-coeffecient and stiffness of human head
- m_u, C_u, k_u - mass, damping-coeffecient and stiffness of upper torso
- C_{uv}, k_{uv} - damping-coeffecient and stiffness between upper torso and viscera
- m_v - mass of viscera
- C_{vl}, k_{vl} - damping-coeffecient and stiffness between viscera and lower torso
- m_l, C_l, k_l - mass, damping-coeffecient and stiffness of lower torso
- m_s, C_s, k_s - mass, damping-coeffecient and stiffness of tractor seat
- m_b - mass of vehicle body
- m_r, C_{cr}, k_{cr} - mass, damping-coeffecient and stiffness of rear tyre
- m_f, C_{cf}, k_{cf} - mass, damping-coeffecient and stiffness of front tyre
- m_{ra},m_{rb},m_{fa},m_{fb} – mass of rear-right,rear-left,front-right and front-left tyres
- k_{ra},k_{rb},k_{fa},k_{fb} – stiffness of rear-right,rear-left,front-right and front-left tyres
- C_{ra},C_{rb},C_{fa},C_{fb} – damping co-effecient of rear-right,rear-left,front-right and front-left tyres
- a -Distance of rear tyre's C.G from C.G of vehicle in x-direction
- b - Distance of front tyre's C.G from C.G of vehicle in x-direction
- c - Distance of left-side tyres from C.G. of vehicle in y-direction
- d -Distance of right-side tyres from C.G. of vehicle in y-direction

II. HALF CAR MODEL

A. Mathematical Modelling

A vibratory system is a dynamic one for which the variables such as the excitations (inputs) and responses (outputs) are time dependent. The overall behaviour of the system can be determined by considering a simple model of the complex physical system. In order to simplify the analysis we can reduce the complex system as a lumped parameter system. For our case we considered an 8 degrees of freedom tractor occupant model (see Fig. 1). This combines 4 DOF half car tractor model and 4 DOF Human biomechanical model. Four degrees of freedom half car tractor model can be further expressed as chassis, front tyre, rear tyre and seat. Similarly four degrees of freedom human model can be expressed as head, upper torso, lower torso and viscera. The 4 DOF human model was selected based on a study performed by Cho-Chung Liang [8] who compared the different degree of freedom systems and concluded that the results of 4 Degrees of freedom assumed by Wan and Schimmels align perfectly with the experimental observations of the human body analysis.

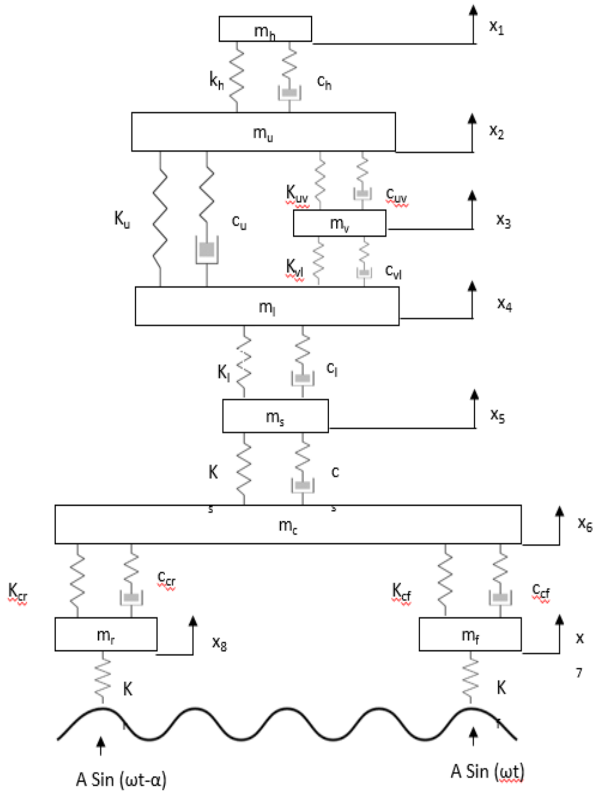

Fig 1 Mathematical Model of tractor occupant model

B. Equations of Motion(Half Car Model)

Once the mathematical model is obtained we use the principles of dynamics and derive the equations that describe the vibration of the system. The equations of motion can be derived conveniently by drawing the free-body diagrams of all the masses involved. The free-body diagram of a mass for the tractor occupant model discussed above is used to derive the equations of motion for each mass separately. So totally we get eight equations considering all the masses. These equations are discussed below:

$$m_h (\ddot{y}_1)+ C_h(\dot{y}_1 - \dot{y}_2)+ k_h(y_1-y_2)=0 \quad \ldots(1)$$

$$m_u (\ddot{y}_2)+ C_h(\dot{y}_2 - \dot{y}_1)+ C_{uv}(\dot{y}_2 - \dot{y}_3)+ C_u(\dot{y}_2 - \dot{y}_4)$$
$$+k_h(y_2-y_1)+ k_{uv}(y_2-y_3)+ k_u(y_2-y_4)=0 \quad \ldots.(2)$$

$$m_v (\ddot{y}_3)+ C_{uv}(\dot{y}_3 - \dot{y}_2)+ C_{vl}(\dot{y}_3 - \dot{y}_4)+ k_{uv}(y_3-y_2)$$
$$+ k_u(y_3-y_4)=0 \quad .....(3)$$

$$m_l (\ddot{y}_4)+ C_u(\dot{y}_4 - \dot{y}_2)+ C_{vl}(\dot{y}_4 - \dot{y}_3)+ C_l(\dot{y}_4 - \dot{y}_5)$$
$$+k_u(y_4-y_2)+ k_{vl}(y_4-y_3)+ k_l(y_4-y_5)=0 \quad \ldots(4)$$

$$m_s (\ddot{y}_5)+ C_l(\dot{y}_5 - \dot{y}_4)+ C_s(\dot{y}_5 - \dot{y}_6)+ k_l(y_5-y_4)$$
$$+ k_s(y_5-y_6) =0 \quad \ldots(5)$$

$$m_l(\ddot{y}_6)+C_s(\dot{y}_6-\dot{y}_5)+C_{cr}(\dot{y}_6-\dot{y}_8)+C_{cf}(\dot{y}_6-\dot{y}_7)$$
$$+k_s(y_6-y_5)+k_{cr}(y_6-y_8)+k_{cf}(y_6-y_7)=0 \quad \ldots(6)$$

$$m_f(\ddot{y}_7)+C_{cf}(\dot{y}_7-\dot{y}_6)++k_{cf}(y_7-y_6)+k_f(y_7)=$$
$$k_f(A)\sin(\omega t) \quad \ldots(7)$$

$$m_r(\ddot{y}_8)+C_{cr}(\dot{y}_8-\dot{y}_6)+k_{cr}(y_8-y_6)+k_r(y_8)=$$
$$A\sin(\omega t-\alpha) \quad \ldots(8)$$

C. System Modler Simulation

To ascertain the riding comfort of a tractor operator the developed 8-DOF half car model was solved using Wolfram SystemModler software. This software has sophisticated tools that aids the design the model (as shown in Fig. 2).

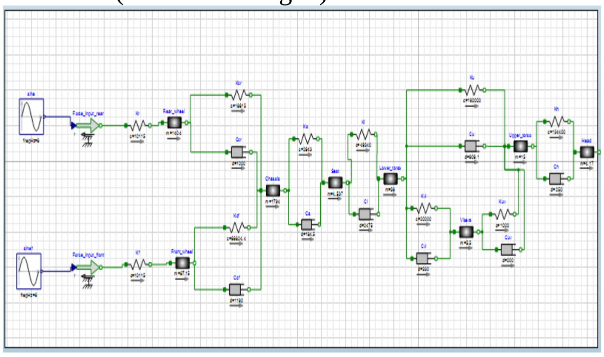

Fig 2 System Modler simulation of half car model

The values for the tractor parameters were adopted from the work carried out by W.Gao [10] and for human biomechanical model was adopted from study performed by Cho-Chung Liang [8]. After substituting the values for mass, stiffness and damping co-efficient (from Table 1) for individual masses the system was simulated by giving periodic road excitation value as adopted from work done by Mothiram K.Patil [11].The simulation was carried for a time period of 120 seconds and the vibration response for each masses were observed as acceleration versus time plot.

Table 1 - The mean values of vehicle system parameters for the half-car model

Component	**Mass (kg)**	**Spring Stiffness (N/m)**	**Damping Co-efficient (Ns/m)**
Head	m_h =4.17	k_h =134400	C_h =250
Upper Torso	m_u =15	k_u =192000	C_u =909.1
Viscera	m_v =5.15	k_{uv} =10000	C_{uv} =200
		k_{vl} =20000	C_{vl} =330
Lower Torso	m_l =36	k_l =49340	C_l =2475
Seat	m_s =4.537	k_s =2943	C_s =184.8
Chassis	m_l =1794	k_{cf} =66824.4	C_{cf} =1190
		k_{cr} =18615	C_{cr} =1000
Tyre	m_f =87.15	k_f =101115	-
	m_r =140.4	k_r =87.15	

D.*Simulation results*

Once the half car model is modelled it is then simulated for 120 seconds to check the response of individual masses and R.M.S values of these responses are tabulated below (see table 2).

Table 2 Simulated R.M.S values of individual masses subjected to periodic excitation for a period of 120 seconds

Description	**Acceleration(A(T)) m/s^2**
Head	0.24
Upper Torso	0.30
Viscera	0.31
Lower Torso	0.53
Seat	0.86
Vehicle body	1.77
Front tyre	2.37
Rear tyre	1.58

III. FULL CAR MODEL

Full car model gives the insight of how the vehicle actually behaves to the road excitation as it takes into account the pitch and roll vibration in addition to vertical vibration.So a 14- DOF full car model of tractor occupant system was modelled and its equations of motion are arrived.

A. Mathematical Model

A 14-DOF full car model of tractor occupant model subjected to periodic excitation is modelled (see fig 3).The 4 DOF human biomechanical model is adopted based on a study performed by Cho-Chung Liang [8] as considered in half car model.The 8 DOF tractor model includes vertical vibrations of each tyres, vehicle body,seat and in addition to it the pitch and roll vibrations of vehicle body too is considered.

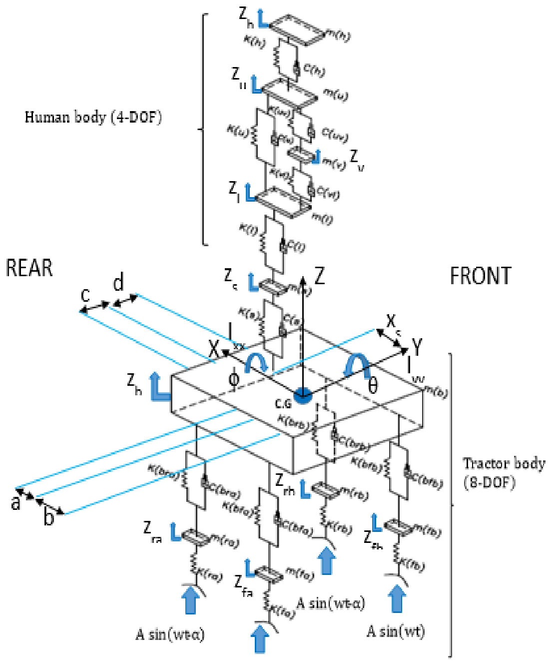

Fig 3 Full Car Model of Tractor occupant system

B. Equations of motion

Once the mathematical model is complete equations of motion of individual masses are derived.These equations of motion are mentioned in the equations 9 to 8 defined below.

<u>Head</u>

$$m_h\ddot{z}_h + k_h(z_h - z_u - x_h\theta - y_h\emptyset) + c_h(\dot{z}_h - \dot{z}_u - x_h\dot{\theta} - y_h\dot{\emptyset}) = 0$$

........(9)

<u>Upper Torso</u>

$$m_u\ddot{z}_u + k_u(z_u - z_l - x_u\theta - y_u\emptyset) + c_u(\dot{z}_u - \dot{z}_u - x_u\dot{\theta} - y_u\dot{\emptyset}) + k_{uv}(z_u - z_v - x_v\theta - y_v\emptyset) + c_{uv}(\dot{z}_u - \dot{z}_v - x_v\dot{\theta} - y_v\dot{\emptyset}) - k_h(z_h - z_u - x_h\theta - y_h\emptyset) - c_h(\dot{z}_h - \dot{z}_u - x_h\dot{\theta} - y_h\dot{\emptyset}) = 0$$

........(10)

<u>Viscera</u>

$$m_v\ddot{z}_v + k_{vl}(z_v - z_l - x_v\theta - y_v\emptyset) + c_{vl}(\dot{z}_v - \dot{z}_l - x_v\dot{\theta} - y_v\dot{\emptyset}) - k_{uv}(z_u - z_v - x_v\theta - y_v\emptyset) - c_{uv}(\dot{z}_u - \dot{z}_v - x_v\dot{\theta} - y_v\dot{\emptyset}) = 0$$

.....(11)

<u>Lower Torso</u>

$$m_l\ddot{z}_l + k_l(z_l - z_s - x_l\theta - y_l\emptyset) + c_l(\dot{z}_l - \dot{z}_s - x_l\dot{\theta} - y_l\dot{\emptyset}) - k_u(z_u - z_l - x_u\theta - y_u\emptyset) - c_u(\dot{z}_u - \dot{z}_l - x_u\dot{\theta} - y_u\dot{\emptyset}) - k_{vl}(z_v - z_l - x_v\theta - y_v\emptyset) - c_{vl}(\dot{z}_v - \dot{z}_l - x_v\dot{\theta} - y_v\dot{\emptyset}) = 0$$

.....(12)

<u>Seat</u>

$$m_s\ddot{z}_s + k_s(z_s - z_b - x_s\theta - y_s\emptyset) + c_s(\dot{z}_s - \dot{z}_b - x_s\dot{\theta} - y_s\dot{\emptyset}) - k_l(z_l - z_s - x_l\theta - y_l\emptyset) - c_l(\dot{z}_l - \dot{z}_s - x_l\dot{\theta} - y_l\dot{\emptyset}) = 0$$

.....(13)

<u>Vehicle Body</u>

Considering vertical vibration

$$m_b\ddot{z}_b + k_{bra}(z_b - a\theta + d\emptyset - z_{ra}) + c_{bra}(\dot{z}_b - a\dot{\theta} + d\dot{\emptyset} - \dot{z}_{ra}) + k_{bfa}(z_b + b\theta + d\emptyset - z_{fa}) + c_{bfa}(\dot{z}_b + b\dot{\theta} + d\dot{\emptyset} - \dot{z}_{fa}) + k_{brb}(z_b - a\theta - c\emptyset - z_{rb}) + c_{brb}(\dot{z}_b - a\dot{\theta} - c\dot{\emptyset} - \dot{z}_{rb}) + k_{bfb}(z_b + b\theta - c\emptyset - z_{fb}) + c_{bfb}(\dot{z}_b + b\dot{\theta} - c\dot{\emptyset} - \dot{z}_{fb}) - k_s(z_s - z_b - x_s\theta - y_s\emptyset) - c_s(\dot{z}_s - \dot{z}_b - x_s\dot{\theta} - y_s\dot{\emptyset}) = 0$$

.....(14)

Considering roll vibration

$$I_{xx}\ddot{\emptyset} + k_{bra}d(z_b - a\theta + d\emptyset - z_{ra}) - c_{bra}d(\dot{z}_b - a\dot{\theta} + d\dot{\emptyset} - \dot{z}_{ra}) + k_{bfa}d(z_b + b\theta + d\emptyset - z_{fa}) + c_{bfa}d(\dot{z}_b + b\dot{\theta} + d\dot{\emptyset} - \dot{z}_{fa}) - k_{brb}c(z_b - a\theta - c\emptyset - z_{rb}) - c_{brb}c(\dot{z}_b - a\dot{\theta} - c\dot{\emptyset} - \dot{z}_{rb}) - k_{bfb}c(z_b + b\theta - c\emptyset - z_{fb}) - c_{bfb}c(\dot{z}_b + b\dot{\theta} - c\dot{\emptyset} - \dot{z}_{fb}) + y_sk_s(z_s - z_b - x_s\theta - y_s\emptyset) + y_sc_s(\dot{z}_s - \dot{z}_b - x_s\dot{\theta} - y_s\dot{\emptyset}) = 0$$

.....(15)

Considering pitch vibration

$$I_{yy}\ddot{\theta} - ak_{bra}d(z_b - a\theta + d\emptyset - z_{ra}) + ac_{bra}d(\dot{z}_b - a\dot{\theta} + d\dot{\emptyset} - \dot{z}_{ra}) + k_{bfa}b(z_b + b\theta + d\emptyset - z_{fa}) + c_{bfa}b(\dot{z}_b + b\dot{\theta} + d\dot{\emptyset} - \dot{z}_{fa}) - k_{brb}a(z_b - a\theta - c\emptyset - z_{rb}) - c_{brb}a(\dot{z}_b - a\dot{\theta} - c\dot{\emptyset} - \dot{z}_{rb}) - k_{bfb}b(z_b + b\theta - c\emptyset - z_{fb}) - c_{bfb}b(\dot{z}_b + b\dot{\theta} - c\dot{\emptyset} - \dot{z}_{fb}) + x_sk_s(z_s - z_b - x_s\theta - y_s\emptyset) + y_sc_s(\dot{z}_s - \dot{z}_b - x_s\dot{\theta} - y_s\dot{\emptyset}) = 0$$

....(16)

<u>Rear right wheel</u>

$$m_{ra}\ddot{z}_{ra} - k_{bra}(z_b - a\theta + d\emptyset - z_{ra}) - c_{bra}(\dot{z}_b - a\dot{\theta} + d\dot{\emptyset} - \dot{z}_{ra}) + k_{ra}z_{ra} = A\sin(\omega t - \alpha)$$

.....(17)

<u>Front right wheel</u>

$$m_{fa}\ddot{z}_{fa} - k_{bfa}(z_b + b\theta + d\emptyset - z_{fa}) - c_{bfa}(\dot{z}_b + b\dot{\theta} + d\dot{\emptyset} - \dot{z}_{fa}) + k_{fa}z_{fa} = A\sin(\omega t)$$

....(18)

<u>Rear left wheel</u>

$$m_{rb}\ddot{z}_{rb} - k_{brb}(z_b - a\theta - c\emptyset - z_{rb}) - c_{brb}(\dot{z}_b - a\dot{\theta} - c\dot{\emptyset} - \dot{z}_{rb}) + k_{rb}z_{rb} = A\sin(\omega t - \alpha)$$

...(19)

<u>Front left wheel</u>

$$m_{fb}\ddot{z}_{fb} - k_{bfb}(z_b + b\theta - c\emptyset - z_{fb}) - c_{bfb}(\dot{z}_b + b\dot{\theta} - c\dot{\emptyset} - \dot{z}_{fb}) + k_{fb}z_{fb} = A\sin(\omega t) \;....(12)$$

....(20)

The following table gives the values of the distance by which each masses are separated from the reference point.This information is required to calculate the Centre of gravity of entire vehicle(see table 3).

Table 3 Distance of C.G.'s of different masses from reference point

Description	Distance of C.G's in X-direction from the reference point (mm)	Distance of C.G's in Y-direction from the reference point (mm)	Distance of C.G's in Z-direction from the reference point (mm)
Mass of rear right tyre(m_{ra})	X_{ra}=355.6	Y_{ra}=27.5	Z_{ra}=355.6
Mass of rear left tyre(m_{rb})	X_{rb}=355.6	Y_{rb}=1997.5	Z_{rb}=355.6
Mass of front right tyre(m_{fa})	X_{fa}=2265.6	Y_{fa}=227.5	Z_{fa}=203

Mass of front left tyre(m_{fb})	X_{fb}=2265.6	Y_{fb}=1997.5	Z_{fb}=203
Mass of vehicle body (m_x)	X_x=1630	Y_x=1340	Z_x=1360
Mass of seat(m_s)	X_s=380.3	Y_s=1340	Z_s=1300

Based on these values the Centre of gravity of entire vehicle is calculated using the conventional formula .The Centre of gravity of entire vehicle in x,y and z direction are 1566.23,1314.5 and 1240.5 respectively.

These values are required to determine the values of a,b, c and d which are distance of respective tyres from C.G of the vehicle.

a =1210.63 mm

b =699.37 mm

c = 683 mm

d = 1087 mm

x_s = 226.23 mm

Following table gives the values of necessary parameters that are required to be substituted to solve the equations and additional data can be adopted from table 1.

Table 4 Mean values of vehicle system parameters for Full-car model

Component	Mass (kg)	Spring Stiffness (N/m)		Damping Co-efficient (Ns/m)	
Seat	m_s =4.537	k_s =2943		C_s =184.8	
Body	m_b =1794	Front-right	k_{bfa} =33412.2	Front-right	C_{cf} =595
		Front-left	k_{bfb}=33412.2	Front-left	C_{bfb}=595
		Back-right	k_{bra}=9307.5	Back-right	C_{bra}=500
		Back-left	k_{brb} =9307.5	Back-left	C_{brb}=500
Front tyre	m_{fa} =43.575	k_{fa} =50557.5		-	
	m_{fb} =43.575	k_{fb} =50557.5			
Rear tyre	m_{ra}=70.2	k_{ra}=43.575		-	
	m_{rb}=70.2	k_{rb}=43.575			

C. State Space representation

The difficulty experienced in solving system of second order differential equations can be avoided by converting it to first order differential equations.This can be done using state space representation method.

Assume following state variables

Z_h=X_1 $\dot{Z}_h$=X_2 Z_u=X_3 $\dot{Z}_u$=X_4 Z_v=X_5 $\dot{Z}_v$=X_6

Z_l=X_7 $\dot{Z}_l$=X_8 Z_s=X_9 $\dot{Z}_s$=X_{10} Z_b=X_{11} $\dot{Z}_b$=X_{12}

ϕ=X_{13} $\dot{\phi}$ =X_{14} θ=X_{15} $\dot{\theta}$ =X_{16} Z_{ra}=X_{17} $\dot{Z}_{ra}$=X_{18}

Z_{fa}=X_{19} $\dot{Z}_{fa}$=X_{20} Z_{rb}=X_{21} $\dot{Z}_{rb}$ =X_{22} Z_{fb}=X_{23} $\dot{Z}_{fb}$=X_{24}

Substituting the above vairables in equation (9) to (20) and writing the quation in state space matrix form,

$$[\dot{X}] = [A]\,[X] + [B]\,[U] \qquad \text{....(21)}$$

$$[Y] = [C]\,[X] + [D]\,[U] \qquad \text{....(22)}$$

Where,

- A is 24*24, is state matrix
- B is 24*4, is the input matrix
- U is 4*4, is input, function of time
- C is 12*24,is output matrix, a constant
- D is 24*4, is direct transition matrix

A1 = [0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]

A3 = [0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]

A5 = [0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]

A7 = [0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]

A9 = [0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0]

A11 = [0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0]

A13 = [0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0]

A15 = [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0]

A17 = [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0]

A19 = [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0]

A21 = [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0]

A23 = [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]

$$[A] = \begin{bmatrix} A1 \\ A2 \\ A3 \\ A4 \\ A5 \\ A6 \\ A7 \\ A8 \\ A9 \\ A10 \\ A11 \\ A12 \\ A13 \\ A14 \\ A15 \\ A16 \\ A17 \\ A18 \\ A19 \\ A20 \\ A21 \\ A22 \\ A23 \\ A24 \end{bmatrix}$$

$$A2 = \left(-\frac{1}{m_h}\right) \begin{bmatrix} k_h \\ C_h \\ -k_h \\ -C_h \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -Y_h K_h \\ -Y_h C_h \\ -X_h K_h \\ -X_h C_h \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \quad A4 = \left(-\frac{1}{m_u}\right) \begin{bmatrix} -k_h \\ -C_h \\ k_u + k_{uv} + k_h \\ C_u + C_{uv} + C_h \\ k_{uv} \\ C_{uv} \\ -k_u \\ -C_u \\ 0 \\ 0 \\ 0 \\ 0 \\ -Y_u K_u - Y_v K_{uv} + Y_h K_h \\ -Y_u C_u - Y_v C_{uv} + Y_h C_h \\ -X_u K_u - X_v K_{uv} + X_h K_h \\ -X_u C_u - X_v C_{uv} + X_h C_h \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \quad A6 = \left(-\frac{1}{m_v}\right) \begin{bmatrix} 0 \\ 0 \\ -k_{uv} \\ +C_{uv} \\ k_{vl} \\ C_{vl} \\ -k_{vl} \\ -C_{vl} \\ 0 \\ 0 \\ 0 \\ 0 \\ -Y_v K_{vl} + Y_v K_{uv} \\ -Y_v C_{vl} + Y_v C_{uv} \\ -X_v K_{vl} + X_v K_{uv} \\ -X_v C_{vl} + X_v C_{uv} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T$$

$$\begin{bmatrix} 0 \\ 0 \\ -k_u \\ +C_u \\ k_u + k_{vl} \\ C_u + C_{vl} \\ -k_l \\ -C_l \\ 0 \\ 0 \end{bmatrix}^T \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ k_l \\ C_l \\ K_s - K_l \\ C_s - C_l \end{bmatrix}^T$$

$$A18 = \left(-\frac{1}{m_{ra}}\right)\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -k_{bra} \\ -C_{bra} \\ -dK_{bra} \\ -dC_{bra} \\ aK_{bra} \\ aC_{bra} \\ k_{bra}+k_{ra} \\ C_{bra} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \quad A20 = \left(-\frac{1}{m_{fa}}\right)\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -k_{bfa} \\ -C_{bfa} \\ -dK_{bfa} \\ -dC_{bfa} \\ -bK_{bfa} \\ -bC_{bfa} \\ 0 \\ 0 \\ k_{bfa}+k_{fa} \\ C_{bfa} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T$$

$$A12 = \left(-\frac{1}{m_b}\right)\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -k_s \\ -C_s \\ K_{bra}+K_{bfa}+K_{brb}+K_{bfb}+K_s \\ C_{bra}+C_{bfa}+C_{brb}+C_{bfb}+C_s \\ dK_{bra}+dK_{bfa}-cK_{brb}-cK_{bfb}+Y_sK_s \\ dC_{bra}+dC_{bfa}-cC_{brb}-cC_{bfb}+Y_sC_s \\ -aK_{bra}+bK_{bfa}-aK_{brb}+bK_{bfb}+X_sK_s \\ -aC_{bra}+bC_{bfa}-aC_{brb}+bC_{bfb}+X_sC_s \\ -k_{bra} \\ -C_{bra} \\ -k_{bfa} \\ -C_{bfa} \\ -k_{brb} \\ -C_{brb} \\ -k_{bfb} \\ -C_{bfb} \end{bmatrix}^T \quad A14 = \left(-\frac{1}{I_{xx}}\right)\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ k_sY_s \\ C_sY_s \\ d(K_{bra}+K_{bfa})-c(K_{brb}+K_{bfb})-K_sY_s \\ d(C_{bra}+C_{bfa})-c(C_{brb}+C_{bfb})-C_sY_s \\ d^2(K_{bra}+K_{bfa})+c^2(K_{brb}+K_{bfb})-K_sY_s^2 \\ d^2(C_{bra}+C_{bfa})+c^2(C_{brb}+C_{bfb})-C_sY_s^2 \\ -adK_{bra}+bdK_{bfa}+acK_{brb}+bcK_{bfb}-Y_sX_sK_s \\ -adC_{bra}+bdC_{bfa}+acC_{brb}+bcC_{bfb}-Y_sX_sC_s \\ -dk_{bra} \\ -dC_{bra} \\ -dk_{bfa} \\ -dC_{bfa} \\ ck_{brb} \\ cC_{brb} \\ ck_{bfb} \\ cC_{bfb} \end{bmatrix}^T$$

$$A22 = \left(-\frac{1}{m_{rb}}\right)\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -k_{brb} \\ -C_{brb} \\ cK_{brb} \\ cC_{brb} \\ aK_{brb} \\ aC_{brb} \\ 0 \\ 0 \\ 0 \\ 0 \\ k_{brb}+k_{rb} \\ C_{brb} \\ 0 \\ 0 \end{bmatrix}^T \quad A24 = \left(-\frac{1}{m_{fb}}\right)\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -k_{bfb} \\ -C_{bfb} \\ cK_{bfb} \\ cC_{bfb} \\ -bK_{bfb} \\ -bC_{bfb} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ k_{bfb}+k_{fb} \\ C_{bfb} \end{bmatrix}^T$$

$$A16 = \left(-\frac{1}{I_{yy}}\right)\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ k_sX_s \\ C_sX_s \\ -aK_{bra}+bK_{bfa}-aK_{brb}+bK_{bfb}-K_sX_s \\ -aC_{bra}+bC_{bfa}-aC_{brb}+bC_{bfb}-C_sX_s \\ -adK_{bra}+bdK_{bfa}+acK_{brb}-bcK_{bfb}-K_sX_sY_s \\ -adC_{bra}+bdC_{bfa}+acC_{brb}-bcC_{bfb}-C_sX_sY_s \\ a^2(K_{bra}+K_{brb})+b^2(K_{bfa}+K_{bfb})-X_s^2K_s \\ a^2(C_{bra}+C_{brb})+b^2(C_{bfa}+C_{bfb})-X_s^2C_s \\ ak_{bra} \\ aC_{bra} \\ -bk_{bfa} \\ -bC_{bfa} \\ ak_{brb} \\ aC_{brb} \\ -bk_{bfb} \\ -bC_{bfb} \end{bmatrix}^T$$

[B] = [B1 B2 B3 B4]

$$[B1] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ K_{fb}/m_{fb} \end{bmatrix} \quad [B2] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ K_{rb}/m_{rb} \\ 0 \\ 0 \end{bmatrix}$$

$$[B3] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ K_{fa}/m_{fa} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad [B4] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ K_{ra}/m_{ra} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[C]= \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0 \end{bmatrix}$$

These equations on solving using Matlab software will give the response of individual masses which are to be compared with the results obtained from half car model.

IV ISO 2631

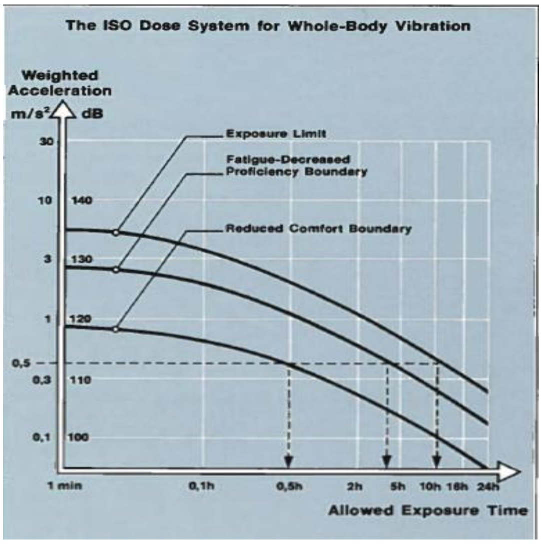

Fig 4 ISO dose system for whole body vibration

International Standards Organization has predefined the limits of vibration expoure that can be felt by the human body[6].It has defined three conditions namely reduced comfort boundary,fatigue decreased proficiency boundary and exposure limits based on the magnitude of vibration response for different exposure time(fig 4) . These conditions clearly indicate that Exposure limit value is more significant with regards to human health as if humans are exposed beyond these values for longer period of time it will have adverse effect on their health.

V CONCLUSION

On comparing the vibration response value (measured at lower torso) obtained from half car model (a_l=0.57 m/s^2) with that of ISO specified values (a_z=0.85 m/s^2) it is clear that the values are well within the limits when simulated for a period of 120 seconds. So simulation of full car model of tractor vehicle occupant can give much better picture of actual vibration response experienced by humans as it takes into account the pitch as well as roll vibrations.Instead of considering periodic excitation the system has to be simulated with random random excitation which gives more accurate results.

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